Here is an expression for the energy levels of an electron in a particular situation (an electron bound to a single proton):

$$E_n = \frac{-13.6 \text{ eV}}{n^2} \qquad n = 1, 2, 3, \cdots \tag{1}$$

1. Sketch the energy level diagram

> **Solution** I'm going to procrastinate putting up a solution to this, since it's a pain to create nice drawings. But you need start by sketching the lowest energy eigenstate and the $E = 0$ axis. Then the next one is $\frac{1}{4}$ of the lowest energy state, and the following is $\frac{1}{9}$ of the lowest energy state, and all the higher ones are crammed together near the $E = 0$ axis.

2. Find at least one allowed transition which produces a photon with a visible color

> **Solution** We can start by looking at the energy of a photon going between any pair of states, and see if it is visible. To find the energy of a photon, we just need to consider conservation of energy. The energy of the photon will be equal to the change in energy of the hydrogen atom. If we start with the $2 \to 1$ transition, we see that
>
> $$E_{\text{photon}} = E_2 - E_1 \tag{2}$$
> $$= 13.6 \text{ eV} \left(1 - \frac{1}{4}\right) \tag{3}$$
> $$\approx 10 \text{ eV} \tag{4}$$
>
> This is *way* behond visible. It has very high energy, or equivalently very short wavelength. From this we can conclude that any transition *to* the $n = 1$ state will not be visible, since any other such transition must result in a higher energy photon.
> We can go backwards and recognize that we need a value of $\frac{1}{n_f^2} - \frac{1}{n_i^2}$ that is around 0.2. Let's try the $n = 3 \to 2$ transition.
>
> $$E_{\text{photon}} = E_3 - E_2 \tag{5}$$
> $$= 13.6 \text{ eV} \left(\frac{1}{4} - \frac{1}{9}\right) \tag{6}$$
> $$\approx 1.9 \text{ eV} \tag{7}$$
>
> Yay, this is squarely in the range of visible light. It is not the only transition we can see, and hence not the only correct answer. But the process of checking is the same.

Note:

$$\lambda = \frac{2\pi\hbar c}{E_{\text{photon}}} \tag{8}$$
$$= \frac{1240 \text{ eV} \cdot \text{nm}}{E_{\text{photon}}} \tag{9}$$

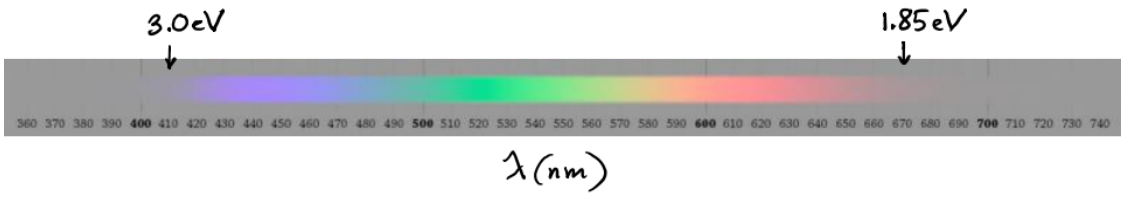


Figure 1: Visible light spectrum