In this unit, you will learn a common technique for solving linear, homogeneous ordinary differential equations that is used when the equation is more complicated than those with constant coefficients. You will start with some simplified pure math examples. Then you will apply the technique to several ordinary differential equations that come up in the solution of the (unperturbed) quantum hydrogen atom (e.g. Legendre's equation, Laguerre's equation).

Motivating Questions

- When and how can you use a power series to solve an ordinary differential equation that you cannot solve in other ways?
- How do you find the coefficients of a power series solution?
- When is an ordinary differential equation an eigenvalue problem and how does that affect the solutions?
- How do the boundary conditions that come from a physical setting affect the power series solutions?

Key Activities/Problems

- `https://paradigms.oregonstate.edu/activities/917`
- `https://paradigms.oregonstate.edu/problem/156`
- `https://paradigms.oregonstate.edu/problem/517`

Unit Learning Outcomes

At the end of this unit, for a linear, homogeneous ODE, you should be able to:

- Use the form of the ordinary differential equation to predict the form and number of power series solutions and their region of convergence.
- Write out the power series solutions for a given ODE in correct form, explaining the meaning of each of the terms.
- For examples, derive a recurrence relation from the ODE.
- For examples, use a recurrence relation to calculate the coefficients of a power series.
- In the case of eigenvalue equations, use the recurrence relation to find eigenvalues that will cause the series to terminate (i.e. have only a finite number of non-zero terms), resulting in polynomial solutions.
- Use a (truncated) power series to approximate the solution(s) to an ODE and discuss where the approximation is valid.