

Equations for any 1-D System with Time-Independent Hamiltonian

Expansion in an Eigenbasis:

$$|\psi\rangle = \sum_m c_m |m\rangle \quad (1)$$

$$c_m = \langle m|\psi\rangle \quad (2)$$

$$\psi(x) = \langle x|\psi\rangle \quad (3)$$

Probabilities:

$$\mathcal{P}_m = |c_m|^2 \quad (4)$$

$$= |\langle m|\psi\rangle|^2 \quad (5)$$

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle \quad (6)$$

$$= \sum_m a_m \mathcal{P}_m \quad (7)$$

$$\Delta \hat{A} = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2} \quad (8)$$

Projection Postulate:

$$|\psi'\rangle = \frac{P_m |\psi\rangle}{\sqrt{\langle \psi | P_m | \psi \rangle}} \quad (9)$$

$$P_m = |m\rangle \langle m| \quad (10)$$

Time Dependence:

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle \quad (11)$$

$$|\psi(t)\rangle = \sum_m c_m |m\rangle e^{-i \frac{E_m}{\hbar} t} \quad (12)$$

Equations Specific to the Ring

Operators and Eigenvalue Equations:

$$L_z \doteq -i\hbar \frac{\partial}{\partial \phi} \quad (13)$$

$$H \doteq -\frac{\hbar^2}{2I} \frac{\partial^2}{\partial \phi^2} \quad (14)$$

$$[H, L_z] = 0 \quad (15)$$

$$L_z |m\rangle = m\hbar |m\rangle \quad (16)$$

$$H |m\rangle = E_m |m\rangle \quad (17)$$

$$= \frac{\hbar^2}{2I} m^2 |m\rangle \quad (18)$$

$$(19)$$

Eigenfunctions:

$$|m\rangle \doteq \frac{1}{\sqrt{2\pi r_0}} e^{im\phi} \quad (20)$$

$$m = \{-\infty, \dots, -1, 0, 1, \dots, \infty\} \quad (21)$$

Inner Product:

$$\langle \psi_1 | \psi_2 \rangle = \int_0^{2\pi} \psi_1^*(\phi) \psi_2(\phi) r_0 d\phi \quad (22)$$