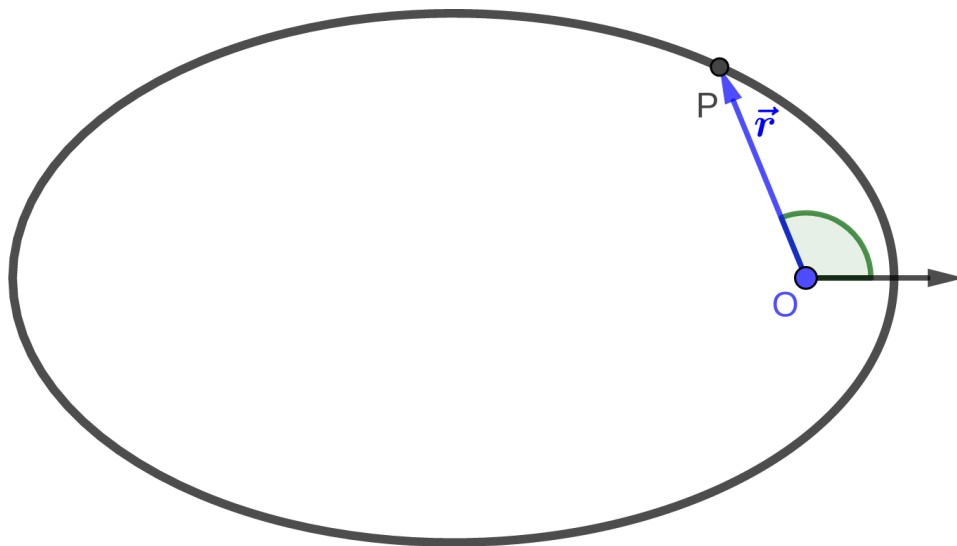
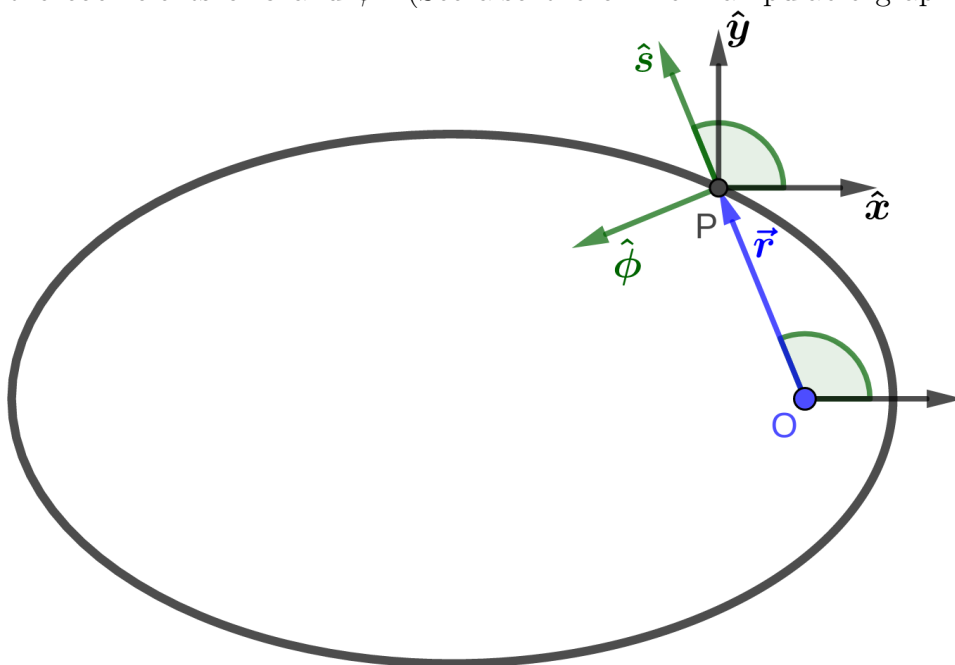


1. On the figure below, draw  $\hat{s}$  and  $\hat{\phi}$  at  $P$ .



2. Find  $\frac{d}{dt}\hat{s}$  and  $\frac{d}{dt}\hat{\phi}$  in terms of  $\hat{s}$  and  $\hat{\phi}$ .

**Solution** First use the figure below to write  $\hat{s}$  and  $\hat{\phi}$  in terms of  $\hat{x}$  and  $\hat{y}$  which do not change with position or time by drawing the four basis vectors all at point  $P$  and reading off the coefficients of  $\hat{s}$  and  $\hat{\phi}$ . (See also the online manipulable graphic version of this figure.)



$$\hat{s} = +\cos \phi \hat{x} + \sin \phi \hat{y}$$

$$\hat{\phi} = -\sin \phi \hat{x} + \cos \phi \hat{y}$$

Then differentiate, using the fact that  $\hat{x}$  and  $\hat{y}$  are constant. Reinterpret the answers using the formulas for  $\hat{r}$  and  $\hat{\phi}$ .

$$\begin{aligned}\dot{\hat{s}} &= -\sin \phi \dot{\phi} \hat{x} + \cos \phi \dot{\phi} \hat{y} \\ &= \dot{\phi} (-\sin \phi \hat{x} + \cos \phi \hat{y}) \\ &= \dot{\phi} \hat{\phi} \\ \dot{\hat{\phi}} &= -\cos \phi \dot{\phi} \hat{x} - \sin \phi \dot{\phi} \hat{y} \\ &= \dot{\phi} (-\cos \phi \hat{x} - \sin \phi \hat{y}) \\ &= -\dot{\phi} \hat{s}\end{aligned}$$

3. Find  $\vec{v}$  in terms of  $\hat{s}$  and  $\hat{\phi}$ .

**Solution**

$$\begin{aligned}\vec{v} &= \frac{d\vec{r}}{dt} \\ &= \frac{d}{dt} s \hat{s} \\ &= \frac{ds}{dt} \hat{s} + s \frac{d\hat{s}}{dt} \\ &= \dot{s} \hat{s} + s \dot{\phi} \hat{\phi}\end{aligned}$$

For more details, please see these lecture notes.