Solve the energy eigenvalue equation for the free particle.

Solution For the Hamiltonian, we only have kinentic energy, so:

$$\hat{H} = T + V = T = \frac{\hat{p}^2}{2m}$$

Then we know $\hat{p} = -i\hbar\frac{\partial}{\partial x}$, so we get:

$$\hat{H} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$$

Now we can find our eigenstates using the energy eigenvalue equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_E(x) = E\psi_E(x) \tag{1}$$

$$\frac{\partial^2}{\partial x^2}\psi_E(x) = -\frac{2mE}{\hbar^2}\psi_E(x) \tag{2}$$

$$\psi_E(x) = \begin{matrix} Ae^{ikx} + Be^{-ikx} \\ \text{or} \\ C\sin kx + D\cos kx \end{matrix} \tag{3}$$

$$\text{where } k^2 = \frac{2mE}{\hbar^2} \tag{4}$$