

You are given the following equations:

$$pV = Nk_B T \quad (1)$$

$$U = \frac{3}{2} Nk_B T \quad (2)$$

$$S = Nk_B \left\{ \ln \left[\frac{V}{N} \left(\frac{mU}{3\pi N \hbar^2} \right)^{3/2} \right] + \frac{5}{2} \right\} \quad (3)$$

The isothermal and isentropic (adiabatic) bulk moduli, β_T and β_S respectively, are defined as follows:

$$\beta_T = -V \left(\frac{\partial p}{\partial V} \right)_T \quad \beta_S = -V \left(\frac{\partial p}{\partial V} \right)_S$$

1. Find an expression for β_T in terms of p , V , and T (at the end, simplify to as few variables as possible)

Solution To find β_T you must evaluate a straightforward partial derivative using the first equation, the ideal gas law.

Divide the left equation ($pV = Nk_B T$) by V to isolate p ;

$$p = \frac{Nk_B T}{V} \quad (4)$$

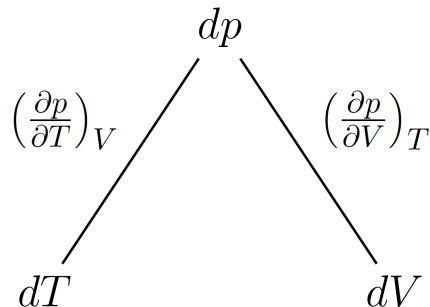
Evaluate the partial derivative;

$$\left(\frac{\partial p}{\partial V} \right)_T = -\frac{Nk_B T}{V^2} \quad (5)$$

Thus,

$$\beta_T = -V \left(\frac{\partial p}{\partial V} \right)_T = \frac{Nk_B T}{V} \quad (6)$$

Note: Alternatively, one can zap the given equation with d , isolate dp , and identify the derivative by comparing the result to the total differential of $p(V, T)$; $dp = \left(\frac{\partial p}{\partial V} \right)_T dV + \left(\frac{\partial p}{\partial T} \right)_V dT$. The total differential can also be represented as a chain rule diagram element (as can any total differential);



2. Find an expression for β_S in terms of p , V , T and S (at the end, simplify to as few variables as possible).

Solution See Ethan's class notes (OneNote file) for a detailed solution.