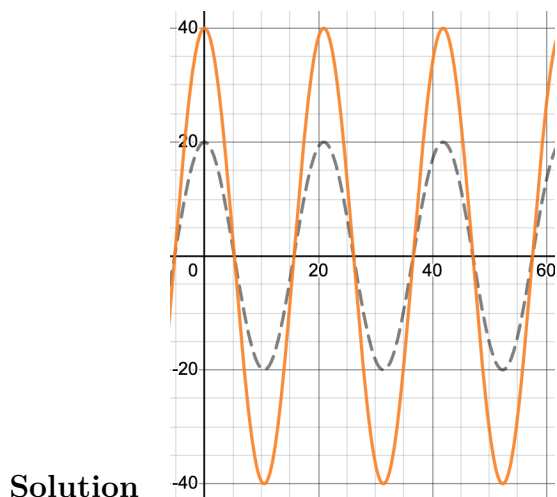


Use the “Masses and Springs” PhET to explore the amplitude and phase parameters of a simple harmonic oscillator.

Attach a 100g block to each spring.

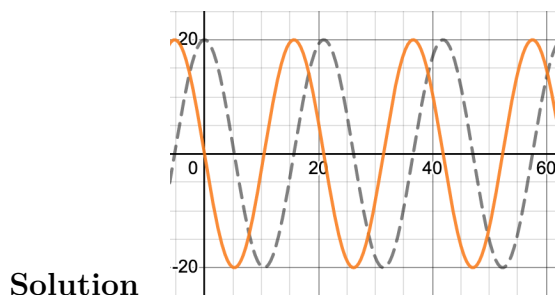
Let the block on the left oscillate with amplitude  $20\text{cm}$ . Then, make it so the right block oscillates with the same frequency  $\omega_0$  but the following parameters:

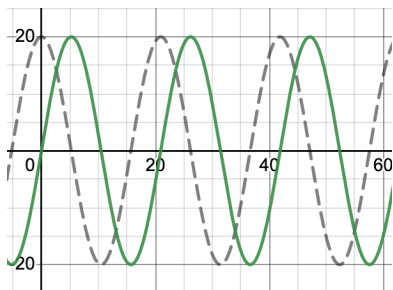
1. Twice the amplitude and relative phase of zero.



$$\begin{aligned} y(t) &= [40 \text{ cm}] \cos \omega_0 t \\ &= [20 \text{ cm}] e^{i\omega_0 t} + [20 \text{ cm}] e^{-i\omega_0 t} \\ &= \text{Re} \left[ [40 \text{ cm}] e^{i\omega_0 t} \right] \end{aligned}$$

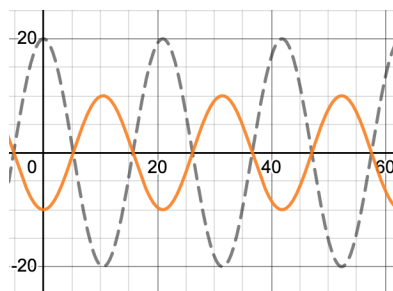
2. Same amplitude and relative phase of  $+\pi/2$ ; also  $-\pi/2$





$$\begin{aligned}
 y(t) &= [20 \text{ cm}] \cos(\omega_0 t \pm \frac{\pi}{2}) \\
 &= \pm [20 \text{ cm}] \sin \omega_0 t \\
 &= [10 \text{ cm}] \cancel{e^{\pm \frac{i\pi}{2}}} e^{\pm i\omega_0 t} + [10 \text{ cm}] \cancel{e^{\mp \frac{i\pi}{2}}} e^{\mp i\omega_0 t} \\
 &= \text{Re} \left[ [20 \text{ cm}] e^{\pm \frac{i\pi}{2}} e^{i\omega_0 t} \right]
 \end{aligned}$$

3. Half the amplitude and relative phase  $+\pi$ ; also  $-\pi$

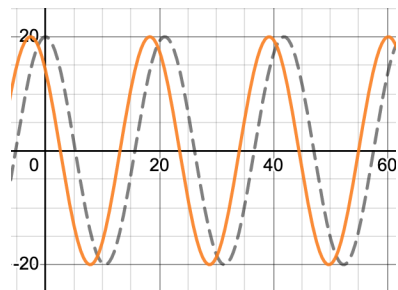


**Solution**

$$\begin{aligned}
 y(t) &= [10 \text{ cm}] \cos(\omega_0 t \pm \pi) \\
 &= \mp [10 \text{ cm}] \cos \omega_0 t \\
 &= [5 \text{ cm}] \cancel{e^{\pm i\pi}} e^{\pm i\omega_0 t} + [5 \text{ cm}] \cancel{e^{\mp i\pi}} e^{\mp i\omega_0 t} \\
 &= \text{Re} \left[ [10 \text{ cm}] \cancel{e^{\pm i\pi}} e^{\pm i\omega_0 t} \right]
 \end{aligned}$$

4. Same amplitude  $A$  and phase  $+\pi/4$ ;

Solution



$$\begin{aligned}
 y(t) &= [20 \text{ cm}] \cos(\omega_0 t + \frac{\pi}{4}) \\
 &= [10\sqrt{2} \text{ cm}] \cos \omega_0 t - [10\sqrt{2} \text{ cm}] \sin \omega_0 t \\
 &= [10 \text{ cm}] e^{\frac{i\pi}{4}} e^{i\omega_0 t} + [10 \text{ cm}] e^{\frac{i\pi}{4}} e^{-i\omega_0 t} \\
 &= \text{Re} \left[ [20 \text{ cm}] e^{\frac{i\pi}{4}} e^{i\omega_0 t} \right]
 \end{aligned}$$

For each situation,

- Plot the position of each block over time. Let the left block be at its maximum displacement at  $t = 0$ .
- Write an equation that describes the motion of the right block.