In this class, we will discuss 4 different forms of the solutions of the equation of motion for a simple harmonic oscillator:

$$\text{A Form: } x(t) = A\cos(\omega t + \phi)$$

$$\text{B Form: } x(t) = B_1\cos(\omega t) + B_2\sin(\omega t)$$

$$\text{C Form: } x(t) = Ce^{i\omega t} + C^* e^{-i\omega t}$$

$$\text{D Form: } x(t) = Re[De^{i\omega t}]$$

1. Relate B_1 & B_2 to A & ϕ

Solution

$$\begin{aligned} A\cos(\omega t + \phi) &= A\cos\omega t\cos\phi - A\sin\omega t\sin\phi \\ &= \underbrace{(A\cos\phi)}_{B_1}\cos\omega t + \underbrace{(-A\sin\phi)}_{B_2}\sin\omega t \end{aligned}$$

2. Relate C to B_1 & B_2

Solution

$$\begin{aligned} Ce^{i\omega t} + C^* e^{i\omega t} &= C[\cos\omega t + i\sin\omega t] + C^*[\cos\omega t + i\sin\omega t] \\ &= \underbrace{(C + C^*)}_{B_1}\cos\omega t + \underbrace{i(C - C^*)}_{B_2}\sin\omega t \end{aligned}$$

3. Relate D to A & ϕ

Solution

$$\begin{aligned} Re[De^{i\omega t}] &= Re[\,|D|e^{i\beta}e^{i\omega t}] \\ &= Re[\,|D|e^{i(\omega t+\beta)}] \\ &= |D|Re[\,e^{i(\omega t+\beta)}] \\ &= |D|\cos(\omega t + \beta) \end{aligned}$$

Comparing to the A form, I see that $|D| = A$ and that $\beta = \phi + 2\pi n$ for integer n

Some useful relations:

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$e^{i\theta} = \cos\theta + i\sin\theta$$