Consider the function:

$$f(x) = \begin{cases} 0 & 0 < x < \frac{2\pi}{3} \\ D & \frac{2\pi}{3} < x < \frac{4\pi}{3} \\ 0 & \frac{4\pi}{3} < x < 2\pi \end{cases}$$

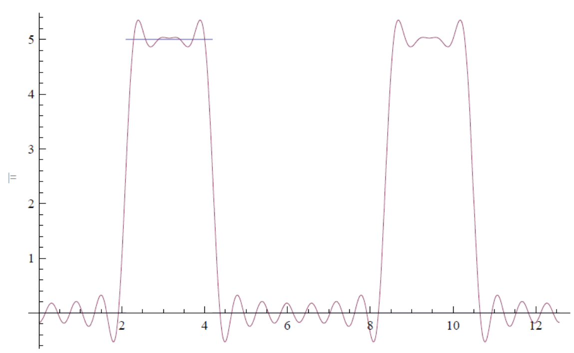
1. You can think of this function as periodic, with period 2π. Represent this function as a Fourier series.

Solution We have been told that this function is periodic, with period 2π, which means that the function is even and only the cosine terms will contribute.
The non-zero coefficients are given by

$$\begin{aligned} a_0 &= \frac{1}{\pi}\int_0^{2\pi} \cos 0\theta\, f(\theta)\, d\theta \\ &= \frac{1}{\pi}\int_{2\pi/3}^{4\pi/3} D\, d\theta \\ &= \frac{D}{\pi}\frac{2\pi}{3} \\ a_n &= \frac{1}{\pi}\int_0^{2\pi} \cos n\theta\, f(\theta)\, d\theta \\ &= \frac{1}{\pi}\int_{2\pi/3}^{4\pi/3} \cos n\theta\, D\, d\theta \\ &= \frac{D}{\pi}\left.\frac{\sin n\theta}{n}\right|_{2\pi/3}^{4\pi/3} \\ &= \frac{D}{\pi}\frac{1}{n}\left[\sin\frac{2\pi n}{3} - \sin\frac{4\pi n}{3}\right] \end{aligned}$$

2. Using your favorite plotting program (or by hand), plot an approximation containing three nonzero terms.

Solution Plotted here is an approximation of two periods of the function, assuming $D = 5$. The approximation contains 8 non-zero terms.



3. Make a chart of your coefficients.

Solution Plotted here is 8 non-zero terms of the chart of the Fourier series.