The formula for the inverse Fourier transform shows that a function $f(x)$ can be written in terms of its Fourier transform via

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(k)\, e^{ikx}\, dk \tag{1}$$

Take the derivative of both sides of this equation with respect to x and simplify. Interpret your expression as the inverse Fourier transform of something.

Solution

$$\begin{aligned}
f(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(k)\, e^{ikx}\, dk & (2)\\
\frac{d}{dx} f(x) &= \frac{d}{dx} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(k)\, e^{ikx}\, dk \right] & (3)\\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(k) \left(\frac{d}{dx} e^{ikx} \right) dk & (4)\\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{f}(k) \left(ik\, e^{ikx} \right) dk & (5)\\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(\tilde{f}(k)\, ik \right) e^{ikx}\, dk & (6)\\
&= \mathcal{F}^{-1} \left(\tilde{f}(k)\, ik \right) & (7)\\
\Rightarrow \mathcal{F} \left(\frac{d}{dx} f(x) \right) &= \tilde{f}(k)\, ik & (8)
\end{aligned}$$

This calculation says that if you know the Fourier transform of a function, then you can find the Fourier transform of the derivative by simply multiplying the Fourier transform of the original function by ik.