

Consider the vector field given by (μ_0 and I are constants): $\vec{B} = \frac{\mu_0 I}{2\pi} \left(\frac{-y\hat{x} + x\hat{y}}{x^2 + y^2} \right) = \frac{\mu_0 I}{2\pi} \frac{\hat{\phi}}{s}$. $\vec{B}$ is the magnetic field around a wire along the z -axis carrying a constant current I in the z -direction.
Ready:

- Determine $\vec{B} \cdot d\vec{r}$ on any radial line of the form $y = mx$, where m is a constant.
- Determine $\vec{B} \cdot d\vec{r}$ on any circle of the form $x^2 + y^2 = a^2$, where a is a constant.
You may wish to express the equations for these curves in polar coordinates.

Solution On a radial line, $d\vec{r} = dr \hat{r}$, so $\vec{B} \cdot d\vec{r} = 0$. On a circle centered at the origin, $d\vec{r} = r d\phi \hat{\phi}$, so $\vec{B} \cdot d\vec{r} = \frac{\mu_0 I}{2\pi} d\phi$

Go: For each of the following curves C_i , evaluate the line integral $\int_{C_i} \vec{B} \cdot d\vec{r}$.

- C_1 , the *top* half of the circle $s = 5$, traversed in a *counterclockwise* direction.

Solution $\frac{\mu_0 I}{2}$

- C_2 , the *top* half of the circle $s = 2$, traversed in a *counterclockwise* direction.

Solution $\frac{\mu_0 I}{2}$

- C_3 , the *top* half of the circle $s = 2$, traversed in a *clockwise* direction.

Solution $-\frac{\mu_0 I}{2}$

- C_4 , the *bottom* half of the circle $s = 2$, traversed in a *clockwise* direction.

Solution $-\frac{\mu_0 I}{2}$

- C_5 , the radial line from $(2, 0)$ to $(5, 0)$.

Solution 0

- C_6 , the radial line from $(-5, 0)$ to $(-2, 0)$.

Solution 0

FOOD FOR THOUGHT

- Construct **closed** curves C_7 and C_8 such that this integral $\int_{C_i} \vec{B} \cdot d\vec{r}$ is nonzero over C_7 and zero over C_8 .
*It is enough to draw your curves; you do **not** need to parameterize them.*

- Ampère's Law says that, for any closed curve C , this integral is (μ_0 times) the current flowing *through* C (in the z direction). Can you use this fact to explain your results to part (a)?
- Is $\vec{B}$ conservative?

Solution Many answers are possible. The integral will be nonzero if and only if the closed path goes around the origin. This vector field represents the magnetic field of a current-carrying wire located along the z axis.

As for whether $\vec{B}$ is conservative, that depends on the allowed domain. It is clearly *not* conservative in the usual sense, since there are closed paths that yield nonzero integrals. But it *does* satisfy the properties of a conservative vector field so long as the domain is restricted to prevent closed paths that go around the origin, such as by removing the positive x axis.