

1. The diagonal of the rectangle on the left below shows (a blown-up picture of) an infinitesimal displacement from the point (x, y) to the nearby point $(x + dx, y + dy)$.

- Label the rectangle with the lengths of the sides.
- Express the sides of the rectangle indicated by arrows as vectors.
Use the unit vectors $\hat{x}$ and $\hat{y}$.
- The diagonal of this rectangle is the vector differential $d\vec{r}$. Express $d\vec{r}$ in terms of $\hat{x}$ and $\hat{y}$.

Solution $d\vec{r} = dx \hat{x} + dy \hat{y}$

- Find the length $ds = |d\vec{r}|$ of the diagonal.

Solution $ds = \sqrt{dx^2 + dy^2}$

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2. The diagonal of the “rectangle” on the right above shows (a blown-up picture of) the *same* infinitesimal displacement, now expressed in polar coordinates, from the point (r, ϕ) to the nearby point $(r + dr, \phi + d\phi)$.

- Label the rectangle with the lengths of the sides. *Careful!*
- Express the sides of the rectangle indicated by arrows as vectors.
Use the natural orthonormal basis defined by the picture, that is, let $\hat{r}$ be the unit vector which points in the direction of increasing r , and let $\hat{\phi}$ be the unit vector which points in the direction of increasing ϕ . Do not attempt to express these vectors in terms of $\hat{x}$ and $\hat{y}$! You do not need to worry about the fact that some sides of the rectangle aren't straight; the rectangle is so small that this error is negligible.
- The diagonal of this rectangle is again the vector differential $d\vec{r}$. Express $d\vec{r}$ in terms of $\hat{r}$ and $\hat{\phi}$.

Solution $d\vec{r} = dr \hat{r} + r d\phi \hat{\phi}$

- Find the length $ds = |d\vec{r}|$ of the diagonal.

Solution $ds = \sqrt{dr^2 + r^2 d\phi^2}$