



In rectangular coordinates, the natural unit vectors are  $\{\hat{x}, \hat{y}\}$ , which point in the direction of increasing  $x$  and  $y$ , respectively. Similarly, in polar coordinates the natural unit vectors are  $\hat{r}$ , which points in the direction of increasing  $r$ , and  $\hat{\phi}$ , which points in the direction of increasing  $\phi$ .

The *unit tangent vector* to a parametric curve is the unit vector tangent to the curve which points in the direction of increasing parameter. The *principal unit normal vector* to a parametric curve is the unit vector perpendicular to the curve “in the direction of bending”, which is the direction of the *derivative* of the unit tangent vector.

1. Consider the parametric curve  $\vec{r} = 3 \cos \phi \hat{x} + 3 \sin \phi \hat{y}$  with  $\phi \in [0, 2\pi]$ . Calculate the unit tangent vector  $\hat{T}$  and the principal unit normal vector  $\hat{N}$  for this curve in terms of  $\hat{x}$  and  $\hat{y}$ .

**Solution**  $\hat{T} = -\sin \phi \hat{x} + \cos \phi \hat{y}$   
 $\hat{N} = -(\cos \phi \hat{x} + \sin \phi \hat{y})$

2. Consider a circle of radius 3 centered at the origin. Determine the unit tangent vector  $\hat{T}$  and the principal unit normal vector  $\hat{N}$  for this curve in terms of  $\hat{r}$  and  $\hat{\phi}$ .

**Solution**  $\hat{T} = \hat{\phi}$   
 $\hat{N} = -\hat{r}$

3. Compare your answers.