Use the uncertainty relation between position and momentum to estimate the energy of the ground state of a particle of mass m subject to a 1-D potential $V(x) = \frac{1}{2}m\omega^2 x^2$.

Solution Starting with the uncertainty relation:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

If I minimize this relation and solve for the uncertainty in the momentum:

$$\Delta p = \frac{\hbar}{2\Delta x}$$

Now, the potential energy varies with x, so I need to minimize the energy with respect to Δx:

$$\begin{aligned} E &= \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 \\ &\approx \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2}m\omega^2 \Delta x^2 \end{aligned}$$

$$0 = \frac{dE}{d\Delta x} = -\frac{\hbar^2}{4m(\Delta x)^3} + m\omega^2 \Delta x$$

$$\Delta x = \sqrt{\frac{\hbar}{2m\omega}}$$

$$\rightarrow E_g = \frac{\hbar^2}{8m}\left(\frac{2m\omega}{\hbar}\right) + \frac{1}{2}m\omega^2\left(\frac{\hbar}{2m\omega}\right)$$

$$\rightarrow E_g = \frac{1}{4}\hbar\omega + \frac{1}{4}\hbar\omega$$

$$E_g = \frac{1}{2}\hbar\omega$$

This turns out to be the same answer you would get when you solve for the energies exactly.