

For the potential well shown in the figure, consider two energy eigenstates, one with energy corresponding to the blue line ($E_1 < V_0$) and one corresponding to the orange line ($E_2 > V_0$).

1. Sketch the wavefunction
2. Identify the classically allowed and forbidden regions
3. Discuss the important qualitative features.

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Solution

Some things to pay attention to:

1. Boundary Conditions:

- a) Both energy eigenstates should go to zero where the potential is infinite, at $x = 0$ and $x = L$.
- b) Both the wavefunction and the derivative of the wavefunction should be continuous at $x = L/2$.

2. Oscillatory vs. Exponential Shape:

- a) In regions where $E > V$, then you have oscillatory solutions.

$$\frac{\partial^2 \phi(x)}{\partial x^2} = -\underbrace{\frac{\sqrt{2m(E-V)}}{\hbar}}_{-k^2} \phi(x)$$

$$\phi(x) = A \sin kx + B \cos kx$$

- b) In regions where $E < V$, then you have decaying exponential solutions.

$$\frac{\partial^2 \phi(x)}{\partial x^2} = -\underbrace{\frac{\sqrt{2m(E-V)}}{\hbar}}_{q^2} \phi(x)$$

$$\phi(x) = A e^{qx} + B e^{-qx}$$

3. Number of Wiggles:

For oscillatory solutions, the wavenumber is related to the difference between the total energy and the potential energy:

$$k = \frac{\sqrt{2m(E-V)}}{\hbar}$$

The larger the difference, the larger the wavenumber. Large wavenumber means a large number of waves per meter (in other words, a lot of wiggles).

4. **Amplitude:** The norm squared of the wavefunction corresponds to the probability density. So, in regions where the particle spends a lot of time, the amplitude should be bigger. The particle spends more time in regions where its kinetic energy is smaller. So, if $E - V$ is smaller, the amplitude should be bigger.