

For the potential well shown in the figure, consider two energy eigenstates, one with energy corresponding to the blue line ( $E_1 < V_0$ ) and one corresponding to the orange line ( $E_2 > V_0$ ).

1. Sketch the wavefunction
2. Identify the classically allowed and forbidden regions
3. Discuss the important qualitative features.

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Some things to pay attention to:

### 1. Boundary Conditions:

- a) Both energy eigenstates should go to zero where the potential is infinite, at  $x = 0$  and  $x = L$ .
- b) Both the wavefunction and the derivative of the wavefunction should be continuous at  $x = L/2$ .

### 2. Oscillatory vs. Exponential Shape:

- a) In regions where  $E > V$ , then you have oscillatory solutions.

$$\frac{\partial^2 \phi(x)}{\partial x^2} = - \underbrace{\frac{\sqrt{2m(E - V)}}{\hbar}}_{-k^2} \phi(x)$$

$$\phi(x) = A \sin kx + B \cos kx$$

- b) In regions where  $E < V$ , then you have decaying exponential solutions.

$$\frac{\partial^2 \phi(x)}{\partial x^2} = - \underbrace{\frac{\sqrt{2m(E - V)}}{\hbar}}_{q^2} \phi(x)$$

$$\phi(x) = Ae^{qx} + Be^{-qx}$$

3. **Number of Wiggles:** For oscillatory solutions, the wavenumber is related to the difference between the total energy and the potential energy:

$$k = \frac{\sqrt{2m(E - V)}}{\hbar}$$

The larger the difference, the larger the wavenumber. Large wavenumber means a large number of waves per meter (in other words, a lot of wiggles).

4. **Amplitude:** The norm squared of the wavefunction corresponds to the probability density. So, in regions where the particle spends a lot of time, the amplitude should be bigger. The particle spends more time in regions where its kinetic energy is smaller. So, if  $E - V$  is smaller, the amplitude should be bigger.