Reflect on the quantum systems you've studied. It turns out, there are only a few kinds of questions that you can ask about a quantum system.

On your whiteboard, brainstorm a list of those questions. What questions can you ask about a quantum system?

Solution A list of Core Quantum Questions:

1. What are the observables? (What quantities can you measure?)

   a) What is the spectrum of values can you get from a set of measurements? (eigenvalues)

   b) What will the state of the system be after a measurement? (eigenvectors)

$$\hat{A}\,|a\rangle = a\,|a\rangle$$

   c) What quantities can you know simultaneously? (commuting observables)

$$0 = [\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

2. What state is the system in?

   a) What is the probability of measuring a particular value of an observable?

$$\begin{aligned}\mathcal{P}(a) &= \Big|\,\langle a|\psi\rangle\,\Big|^2 \\ &\doteq \left|\int_{\text{all space}} \phi_a^*(x)\psi(x)dx\right|^2\end{aligned}$$

   b) What is the probability of measuring a range of values?

$$\mathcal{P}(a_1 \& a_2) = \sum_{m=1}^{2} \Big|\,\langle a_m|\psi\rangle\,\Big|^2$$

$$\mathcal{P}(a \le x \le b) = \int_a^b |\psi(x)|^2 dx$$

   c) What is the expectation value?

$$\begin{aligned}\langle A\rangle &= \sum_m \mathcal{P}_{a_m} a_m \\ &= \langle\psi|A|\psi\rangle \\ &\doteq \int_{\text{all space}} \psi^*(x)\,\hat{A}\,\psi(x)\;dx\end{aligned}$$

d) What is the uncertainty?

$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$$

e) How are uncertainties of different quantities related?

$$\Delta A \Delta B \geq \frac{1}{2} \left| \langle [A, B] \rangle \right|$$

3. What will happen to the state when I make a measurement?

$$|\psi_f\rangle = \frac{\hat{P} |\psi_i\rangle}{\sqrt{\langle \psi_f | \hat{P} |\psi\rangle}}$$

where

$$\hat{P} = \sum_m |a_m\rangle \langle a_m|$$

that correspond to the values a_m that you got from the measurement (Notice the verb tense here. You have to know what the collection of output states is. This is not a deterministic calculation.)

4. How will a quantum state evolve with time?

If $\hat{H} \neq \hat{H}(t)$

$$|\psi(t)\rangle = \sum_n c_n(0)\, e^{-iE_n t/\hbar} |E_n\rangle$$