First and Second Derivatives

In this activity we will bring together many of the concepts that we learned in class so far. Let us think for a moment about the definition of a derivative, before the limit to zero is taken. This is also called a finite difference ratio or a difference quotient. Wikipedia has a good page about it at: https://en.wikipedia.org/wiki/Finite_difference. The derivative reads:

$$\frac{dy}{dx} = \frac{y(x+\Delta x) - y(x)}{\Delta x} \tag{1}$$

Given this definition for a derivative, it is fairly simple to write a python script that computes the first derivative of a tabulated function. The only subtlety to keep in mind is that if the function is tabulated in an array with n elements, the derivative can be calculated only for $n-1$ of them.

Here is an example of such a function:

```
def firstDer(x,y):
    dx = x[1] - x[0]
    derivative = np.zeros(x.size-1)
    for i in range(x.size-1):
        derivative[i]= ( y[i+1] - y[i] ) / dx
    return(derivative)
```

Let us now test this script. Please:

1. Write a script that includes the above first derivative function. The script should create an array of x values, compute $y(x) = 3x$ and compute the derivative. Check that the derivative is indeed 3 for all values of x

2. Modify the script to compute $y(x) = \sin(x)/\sqrt{x}$ and its derivative. Plot both the function and the derivative in a graph. Add labels to the axes and a legend.

3. Challenge activity: re-write the function firstDer in such a way that you do not need a loop in it.

Second derivatives - Let us now add some complexity. Analogously to what we discussed for the first derivative, the second derivative can be calculated as:

$$\frac{d^2y}{dx^2} = \frac{y(x+2\Delta x) - 2y(x+\Delta x) + y(x)}{\Delta x^2} \tag{2}$$

Let us now work on the following activities:

1. Write a python script that computes the second derivative of the function $y(x) = 3x + x^2$. Check that the second derivative is 2 for all values of x.

2. Modify the script to compute the function $y(x) = 3x + x^2 - 0.2x^3$ and plot its values, its first derivative, and its second derivative. Add a legend and label the axes.

3. Challenge activity Try to use the same logic to write a script that computes the integral of a tabulated function by adding up the trapezoids under the curve. Calculate

$$\int_0^{10} \frac{\sin(x)}{\sqrt{x}} dx \tag{3}$$

4. Extra Challenging Activity Plot the curve

$$Y(x) = \int_0^{x} \frac{\sin(x')}{\sqrt{x'}} dx' \tag{4}$$