Before discussing the spectral intensity of blackbodies, I should explain what a **blackbody** is. It is an object that is perfectly black. We can look at the spectra of glowing objects, and it turns out that any object that looks black when cold (and will always absorb all light that hits it) will glow with an identical (continuous) spectrum and with the same intensity. We could prove this using the Second Law of Thermodynamics if we imagine what would happen if we had two blackbodies that glowed different amounts when at the same temperature: the one that glowed more could be used to heat the other one up.

Thus the spectrum of a blackbody is of considerable interest. Some commonly seen objects that glow essentially as blackbodies:

- the sun
- the black space in the sky between the stars and galaxies
- an incandescent light bulb filament
- red hot iron
- charcoal
- a hole in a box
- a black hole

Note also that if a body reflects some fraction of incident light at a given wavelength, then it will radiate proportionately less than than a blackbody radiates at that wavelength. This also can be shown from the Second Law of Thermodynamics.

intensity An **intensity** has dimensions of energy per time per area, and allows us to find how much energy is flowing through a given area in a given amount of time by multiplying by the area and the time.

spectral intensity A **spectral intensity with respect to** x has dimensions of intensity per x, and we can find the intensity (see above) by integrating the spectral intensity with respect to x between a couple of limits.

Spectral intensity with respect to frequency The spectral intensity with respect to frequency of a blackbody is given by

$$S_\omega(\omega) = \frac{\hbar\omega^3}{4\pi^3 c^2} \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} \tag{1}$$

FIXME show that the dimensions work right. If we integrate it $d\omega$, we get energy per time per area, as we expect. Yay.

To find the total intensity of light radiated, we just need to integrate over all possible frequencies. That's a bit of a pain to do analytically (but totally), so let's start by examining the limiting cases.

Limiting cases We can check the limits of this expression to be sure that our toy model agrees.

Low frequency When $\hbar\omega \ll k_B T$ we can use the power series expansion of $e^x = 1 + x + \cdots$ to find

$$S_\omega(\omega) \approx \frac{\hbar\omega^3}{4\pi^3 c^2}\frac{1}{\cancel{1} + \frac{\hbar\omega}{k_B T} + \cdots - \cancel{1}} \tag{2}$$

$$= \frac{\hbar\omega^3}{4\pi^3 c^2}\frac{k_B T}{\hbar\omega} \tag{3}$$

$$= \frac{\omega^2 k_B T}{4\pi^3 c^2} \tag{4}$$

where you can see that Plank's constant disappears, which is good because this is the classical limit. It also scales the same as our toy model. They differ in overall constant, because this is looking at a black material in a per-area way, where our toy model was giving the total energy radiated by a single oscillating charge.

High frequency When $\hbar\omega \gg k_B T$ we can't use a power series to approximate the exponential (since the thing in the exponential is large, not small), but we *can* recognize that the exponential is way bigger than 1.

$$S_\omega(\omega) = \frac{\hbar\omega^3}{4\pi^3 c^2}\frac{1}{e^{\frac{\hbar\omega}{k_B T}} - \cancelto{0}{1}} \tag{5}$$

$$\approx \frac{\hbar\omega^3}{4\pi^3 c^2} e^{-\frac{\hbar\omega}{k_B T}} \tag{6}$$

which drops off exponentially at high frequencies, as we predicted.

Sketching the spectral intensity Now that we have the limiting cases, you can go ahead and sketch on a single set of axes the spectral intensity at two temperatures differing by a factor of two. See `https://paradigms.oregonstate.edu/activity/862/`.

Spectral intensity with respect to wavelength The spectral intensity can also be expressed with respect to wavelength:

$$S_\lambda(\lambda) = \frac{2\pi h c^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1} \tag{7}$$

Note *The change of variables for a distribution function must be done carefully. I haven't shown you the details of how to do this.*

Note that S_ω and S_λ have different dimensions, and different meanings. Moreover, the question "what color of light has the peak intensity" does not have a unique answer, which is kind of annoying. The color that we see certainly does not correspond precisely to either peak spectral intensity, since our eyes have a sensitivity that depends on frequency (or lambda) in a complicated way. All that said, the two peaks of spectral intensity are not that far off, and certainly do give us a ballpark estimate of what color the glowing object will have.

0.1 Spectral distribution of thermal radiation intensity

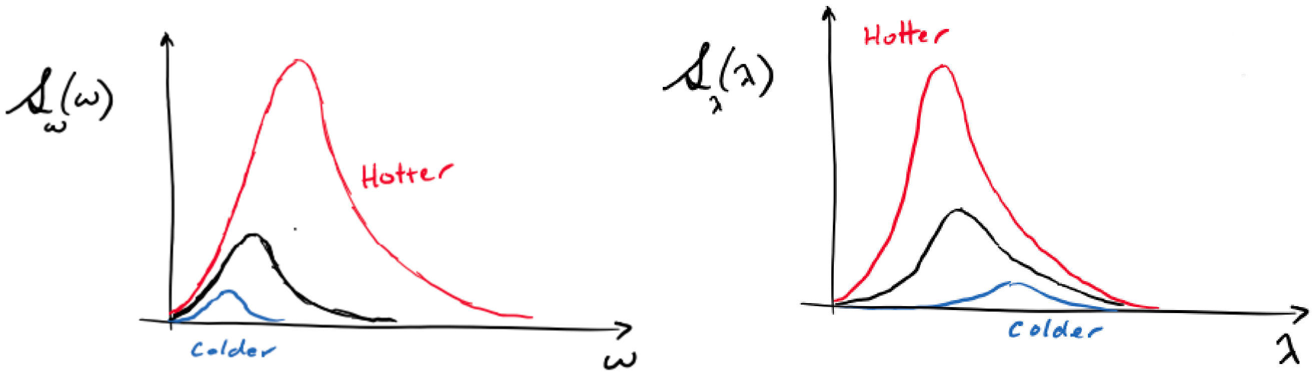

Figure 1: At hotter temperatures, the peak moves to higher frequencies and lower wavelength. At every frequency (or wavelength) there is greater intensity at hotter temperatures.

For most calculations in this class, we will use S_λ.