Nuclear energy (fission or fusion) could save us. Nuclear bombs could destroy us. It is an area of physics that remains a contemporary challenge.

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Figure 1: Recent articles address the debate regarding the role of nuclear power in addressing climate change. Nuclear power does not produce greenhouse gasses, and nuclear reactors can adjust their output to meet high demand. But dealing with radioactive waste remains a political challenge. NuScale is a company based in Oregon (with a Corvallis office!) which is one of several companies working to develop and market modular nuclear reactors that are small, safe and economical.

Let's consider the following event:

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n} \qquad (1) $$

In this scenario a uranium nucleus ${}^{235}_{92}\mathrm{U}$ (which consists of 92 protons and 143 neutrons) gets hit by a slow neutron (${}^{1}_{0}\mathrm{n}$) and splits apart, creating a barium nucleus ${}^{141}_{56}\mathrm{Ba}$ (with 56 protons and 95 neutrons), a krypton nucleus (with 36 protons and 56 neutrons) and three neutrons, which taken together have an awful lot of energy.

This reaction is at the core of many nuclear power plants, and we'll start by analyzing this fission process, before ending with you looking at fusion.

0.1 Mass-energy relationship

Mass turns out to be one form of energy. The energy for a given mass is

$$ E = mc^2 \qquad (2) $$

(Check dimensions: mass times speed^2 is in fact energy)

So if we want to know how much useful energy can be generated from a given nuclear reaction, we can just compare masses of the initial and final products.

FIXME add energy flow diagram for fission of U 235

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}\mathrm{n} \qquad (3) $$

isotope	mass (u)	mc^2 (J)
${}^{235}_{92}\mathrm{U}$	235.044	35.0784×10^{-9}
${}^{92}_{36}\mathrm{Kr}$	91.926 u	13.7192×10^{-9}
${}^{141}_{56}\mathrm{Ba}$	140.914	21.0303×10^{-9}
${}^{1}_{0}\mathrm{n}$	1.009 u	0.1506×10^{-9}

Now if we add up the energies of the ${}^{141}_{56}\mathrm{Ba}$ and ${}^{92}_{36}\mathrm{Kr}$ and two neutrons, we get 35.0507×10^{-9} J. This is smaller than the mass of the original uranium by 2.77×10^{-11} J. There are also several other fission paths, see Wikipedia article which average out to produce 3.24×10^{-11} J per ${}^{235}_{92}\mathrm{U}$. By measuring the masses of atoms, we can determine both whether they will split apart, and how much energy will be released when they do split apart.

0.2 Is this a lot of energy?

3×10^{-11} J doesn't sound like much energy. But let's ask ourselves how long your weight in uranium would be able to provide the United States' electricity needs.

The mass of a ${}^{235}_{92}$U is 2.2×10^{-25} kg, and the electrical power use of the U.S.A is about 10^{12} J. The average mass of a person is about 80 kg. So

$$\text{time it powers us} = \frac{3 \times 10^{-11}\ \text{J/atom}}{10^{12}\ \text{J/s}} \frac{80\ \text{kg}}{2.2 \times 10^{-25}\ \text{kg/atom}} \tag{4}$$

$$\approx 1\ \text{day} \tag{5}$$

So we would need one person's weight in uranium per day to provide the electricity needs of the United States, if you had a 100% efficient nuclear power plant. There is indeed a significant amount of energy stored in a uranium nucleus. For comparison, the same amount of energy would require 4×10^9 kg of coal, which is more than I can carry.

0.3 What is the size of a nucleus?

Nuclear forces are extremely short-range, and roughly speaking happen only when nucleons "touch". So we can estimate the size of a nucleus by assuming that all of the energy that is released in this fission reaction exists as electrostatic energy when a Barium and a Krypton touch.

The size of a nucleus is related to how much electrostatic potential energy is stored in it. When you bring two charges close together

$$U_{\text{elec}} = k_C \frac{q_1 q_2}{r} \tag{6}$$

(if the forces are $\propto 1/r^2$ then the potential energy $\propto 1/r$)

FIXME add plot of U vs r

Let's consider a ${}^{235}_{92}$U nucleus breaking apart into a Barium nucleus (56 protons) and a Krypton nucleus (36 protons).

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$$3 \times 10^{-11}\ \text{J} \approx k_C \frac{(36e)(56e)}{d_{\text{nucleus}}} \tag{7}$$

$$d_{\text{nucleus}} \approx \left[9 \times 10^9 \frac{\text{J} \cdot \text{m}}{\text{C}^2}\right] \cdot 36 \cdot 56 \frac{\left[1.6 \times 10^{-19}\ \text{C}\right]^2}{3 \times 10^{-11}\ \text{J}} \tag{8}$$

$$\approx 1.5 \times 10^{-14}\ \text{m} = 15\ \text{fm} \tag{9}$$

This is what I expect for the *average diameter* of krypton and barium, so their radius should be about 7 or 8 fm.

How is nuclus size experimentally measured? To measure the size of a nucleus, you can bounce alpha particles ($^4_2$He nuclei) off of it. The first experiment for this used gold foil as the target. On the right you can see data for a lead foil measurement from 1960.

One can predict the distribution of angles they should scatter at if the nuclei are point particles. This prediction matches experiment very well, so long as the kinetic energy of the alpha particles is small enough, but at a certain point the scattering pattern starts to change.

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Figure 2: Experimental differential cross section of alpha particles scattered off of Pb, measured by Eisberg and Porter (1960). The dashed line is the prediction based on the assumption that the nucleus and alpha particle are both point particles, and the dots are experimental measurements.

Assume that the energy at which the curve deviates corresponds to the energy required to make the nuclei touch. What does this make the sum of the radius of the lead nucleus plus the alpha particle?

0.4 Fusion

Fusion is the nuclear reaction that powers the sun. It is also the reaction that many people dream of controlling on Earth for electrical energy generation. It also powers hydrogren bombs, which is somewhat less environmentally friendly.