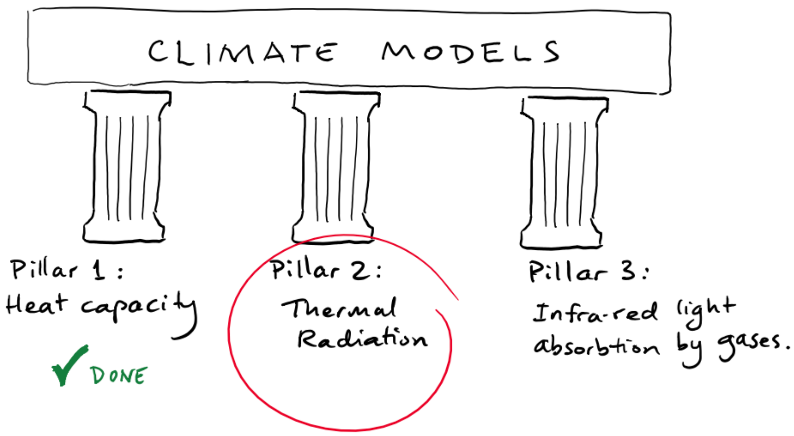



Thermal radiation is also commonly known as **blackbody radiation**, although technically blackbody radiation means the thermal radiation of an object that is black. Hot objects will glow visibly, but even cold objects glow in the infrared spectrum or evn colder objects would glow with microwave or radiofrequency radiation.

Figure 1: This metal visibly glows almost white because it is hot and $k_B T \approx 2$ eV. You can see that parts of it which are a bit cooler glow red, and other parts don't glow at all in the visible spectrum.

# 1 Toy model for thermal radiation

As we have discussed before, light comes from accelerating charges. To get light of a fixed color, a charge should (classically) be shaken with a fixed frequency, which suggests that we consider as a toy model a material containing charges on springs. Since a charge on a spring oscillates with a fixed frequency (regardless of amplitude), it will radiate light that itself has a fixed frequency.

The key classical physics required for thermal radiation is how a charged particle radiates light. If a particle is oscillating with a fixed frequency, the frequency of the light is known, but we still need to know the rate at which energy is radiated. In classical physics this is given by the Larmor formula, which relates the rate at which energy is radiated to the acceleration and charge of a particle:

$$\text{Energy rate} = \frac{2}{3}\frac{q^2 a^2}{4\pi\epsilon_0 c^3} \qquad \text{(in SI units)} \tag{1}$$

where $q$ is the charge and $a$ is its acceleration. The point is that the rate at which energy is radiated is proportional to the acceleration *squared*. We will get back to this.

Now let's introduce a toy model (which could theoretically be built using nanotechnology) that will allow charged masses on sprints to vibrate and emit thermal radiation.

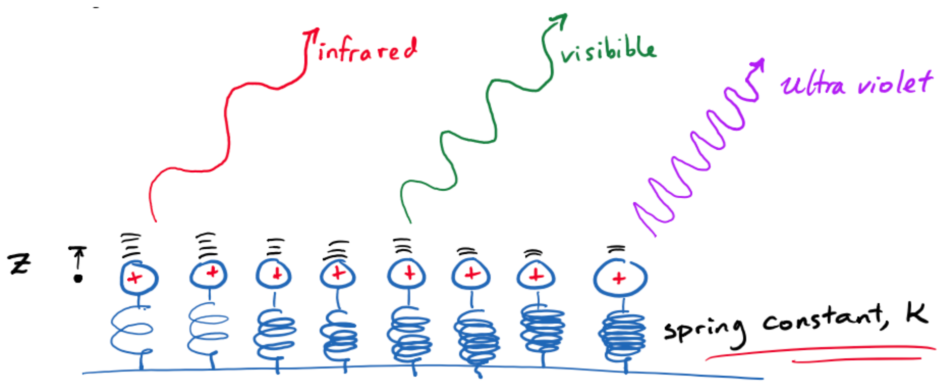



Figure 2: This is a model for a "blackbody" because it can absorb light of any frequency (if we have enough springs). We use the Greek letter $\kappa$ ('kappa') for the spring constant to reduce the risk of confusion with Boltzmann's constant $k_B$.

## 1.1 Analyze this physical system

We can pull out of our physics tool kit some old tools and some new:

- Equipartition theorem
- Harmonic oscillator
- Larmor formula (new!)
- Quantum energy levels

If the nanoscale toy is made from 1-dimensional oscillators, the energy of each oscillator can be written as 2 independent quadratic terms:

$$\text{total energy} = \underbrace{\frac{1}{2}mv_z^2}_{\frac{1}{2}k_BT} + \underbrace{\frac{1}{2}\kappa z^2}_{\frac{1}{2}k_BT} \tag{2}$$

where $z$ is the vertical displacement from equilibrium and $v_z$ is the velocity in that direction. From the equipartition theorem I associate $\frac{1}{2}k_BT$ with each quadratic term.

Therefore, the time-averaged value of $z^2$ is given by (note $\langle x \rangle$ indicates an average)

$$\frac{1}{2}\kappa\langle z^2\rangle = \frac{1}{2}k_BT \tag{3}$$

$$\langle z^2\rangle = \frac{k_BT}{\kappa} \tag{4}$$

The root mean squared displacement can be related to the amplitude of oscillation

$$\sqrt{\kappa\langle z^2\rangle} = \sqrt{\frac{k_B T}{\kappa}} = \frac{z_0}{\sqrt{2}} \tag{5}$$

$$z(t) = z_0 \sin\omega_0 t \tag{6}$$

where $\omega_0$ is the resonant frequency $\omega_0 = \sqrt{\kappa/m}$.

Now we can put these results into the Larmor formula for radiation from an accelerating charge.

$$\text{radiated energy rate} = \frac{2}{3}\frac{q^2 a^2}{4\pi\epsilon_0 c^3} \qquad \text{Larmor formula} \tag{7}$$

$$a = \frac{d^2 z}{dt^2} = -\frac{z_0}{\sqrt{2}}\omega_0^2 \sin\omega_0 t \tag{8}$$

$$\text{radiated energy rate} = \frac{2}{3}\frac{q^2}{4\pi\epsilon_0 c^3}\frac{z_0^2\omega_0^4}{2}\sin^2\omega_0 t \tag{9}$$

Now we can use the handy property of sinusoids that the average value of $\sin^2 x = \frac{1}{2}$ to find the average rate of radiation:

$$\langle\text{radiated energy rate}\rangle = \frac{1}{12}\frac{q^2}{4\pi\epsilon_0 c^3}\frac{k_B T}{\kappa}\omega_0^4 \tag{10}$$

$$= \frac{1}{12}\frac{q^2}{4\pi\epsilon_0 c^3}\frac{k_B T}{m\omega_0^2}\omega_0^4 \tag{11}$$

$$= \frac{1}{12}\frac{q^2}{4\pi\epsilon_0 c^3}\frac{k_B T}{m}\omega_0^2 \tag{12}$$

## 1.2 Sense making

- Light radiated from *one* oscialltor increases with $T$.
- The low-frequency oscillators move a bit more, $z_0 \propto 1/\sqrt{\kappa}$, but the high frequency oscillators radiate more energy...

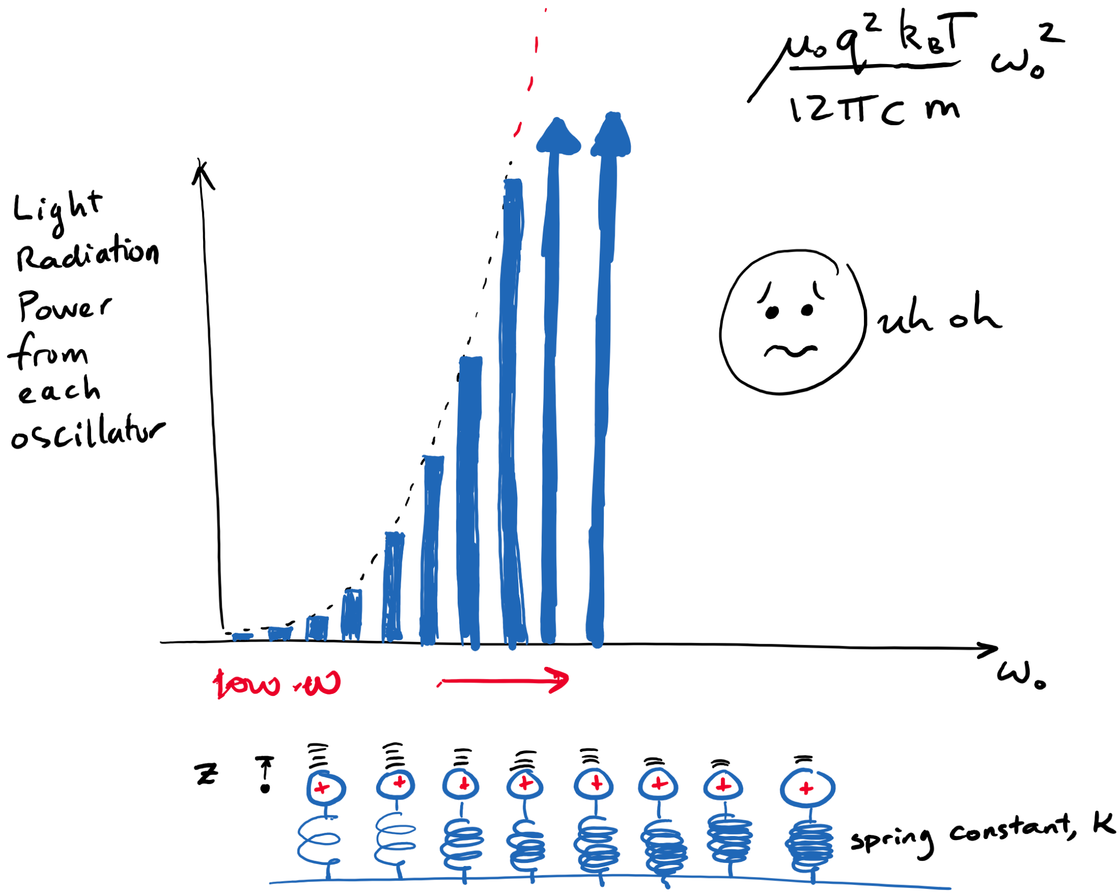



Figure 3: By making springs with very high spring constant, this theory predicts that an unlimited amount of light energy will be radiated. Our toy has turned into a supervillain ultraviolet death ray?

## 1.3 Quantum energy levels to complete the theory

The energy levels of a quantum harmonic oscillator are

$$E_n = \left\{ \frac{1}{2}\hbar\omega_0, 1\frac{1}{2}\hbar\omega_0, 2\frac{1}{2}\hbar\omega_0, 3\frac{1}{2}\hbar\omega_0, 4\frac{1}{2}\hbar\omega_0, \cdots \right\} \tag{13}$$

The energy diffference between adjacent levels is $\hbar\omega_0$, which is the frequency that is radiated (because the oscillator doesn't skip a level when radiating), so our photons have frequency $\omega_0$ and energy $\hbar\omega_0$.

This light emission will only be activated when $k_B T > \hbar\omega_0$, similar to what we saw with the heat capacity.

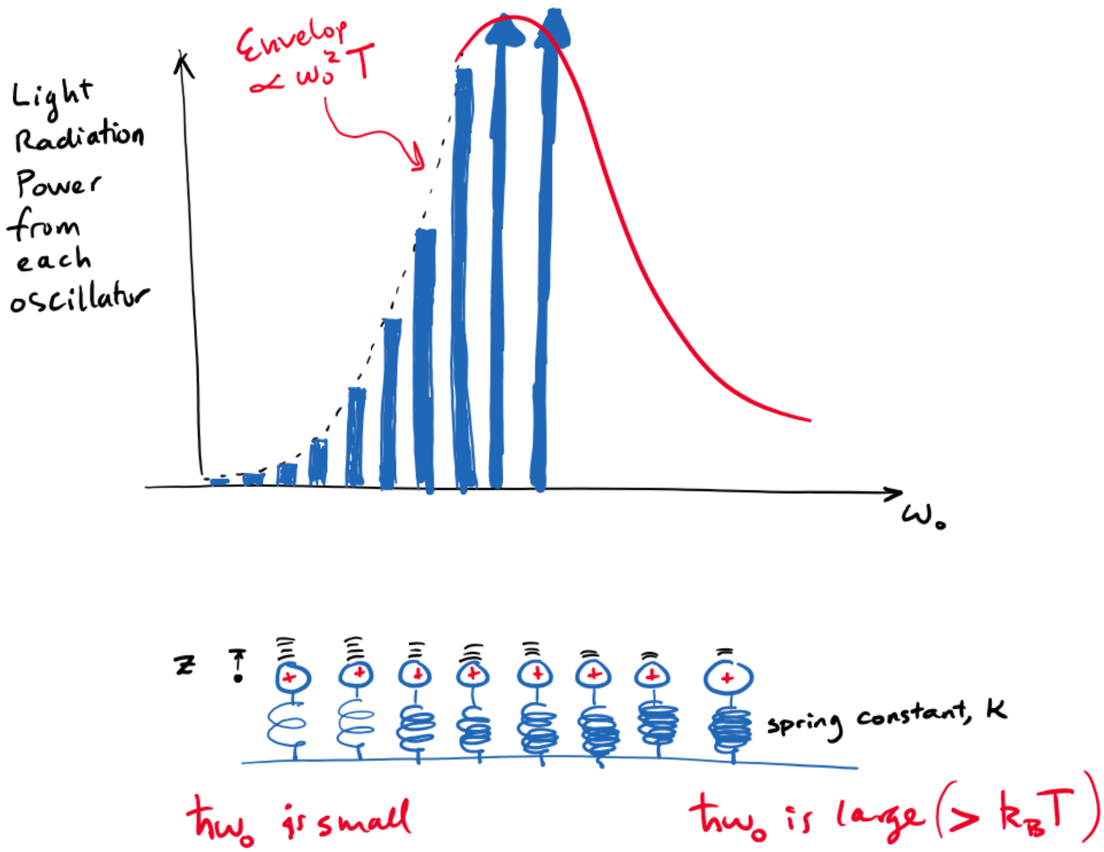
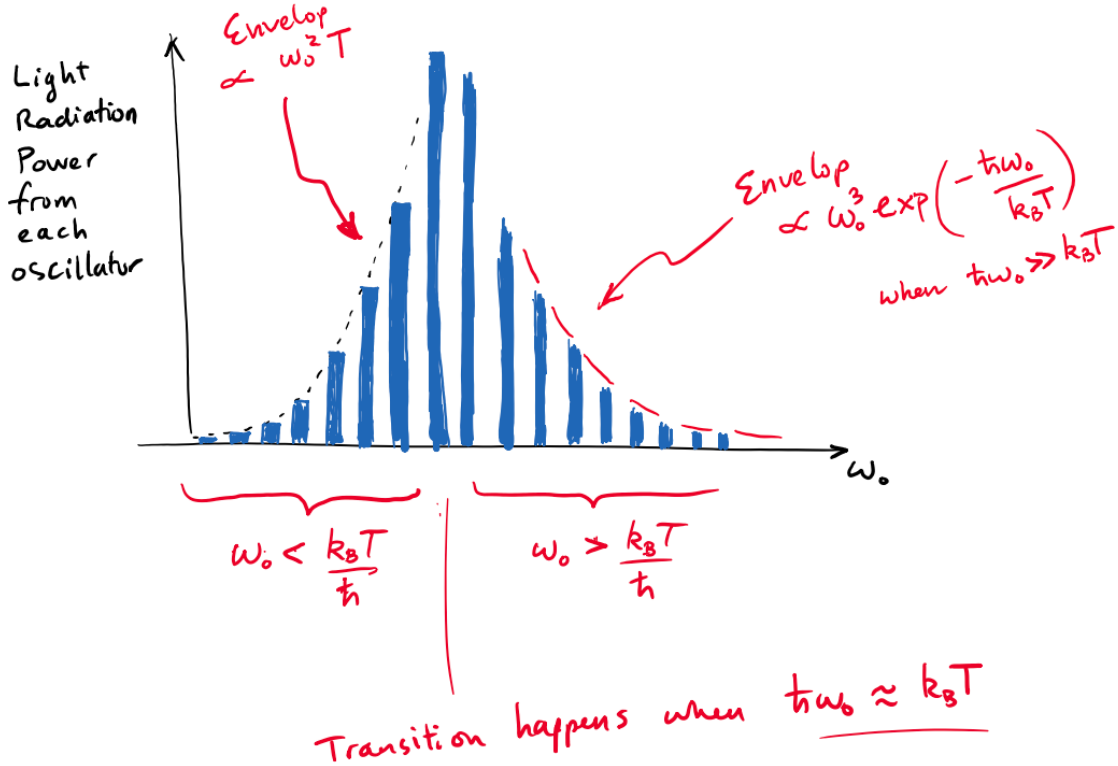



Figure 4: Each oscillator has a different temperature threshold for being "switched on".


Light Radiation Power from each oscillator
Envelop ∝ $\omega_0^2 T$
Envelop ∝ $\omega_0^3 \exp\left(-\frac{\hbar\omega_0}{k_B T}\right)$
when $\hbar\omega_0 \gg k_B T$
$\omega_0$
$\omega_0 < \frac{k_B T}{\hbar}$
$\omega_0 > \frac{k_B T}{\hbar}$
Transition happens when $\hbar\omega_0 \approx k_B T$