Step 1: Quantify the work This requires us to compute force times distance over a single cycle.

$$W = -\int p dV \tag{1}$$

$$= -p\Delta V \qquad \text{if the pressure is constant} \tag{2}$$

We learned this in `https://paradigms.oregonstate.edu/activity/745` and `https://paradigms.oregonstate.edu/activity/746`.

Step 2: Quantify the heat This is what we will do now, and you will finish in `https://paradigms.oregonstate.edu/activity/748`.

Recall the cycle we have already studied:

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Figure 1: Gas engine path

We define the amount of energy transfered by heating (a.k.a. heat) Q to be the amount of energy *added* to the gas by heating. So if the gas heats up its environment, then Q will be negative.

Time period	Description (in words)	W	Q
0-10s	gas is heated at fixed volume	0	?
10-20s	gas expands at fixed pressure	-100 J	?
20-30s	gas is cooled at fixed volume	0	?
30-40s	gas is compressed at fixed pressure	50 J	?

How to figure out Q?

The equation of state Find out the relationship between p, V, and T for this gas.

The internal energy Find out the relationship between p, V, T, and internal energy for this gas. *This is* ***an*** *equation of state.*

Detective work like Sherlock Holmes! *Using the Laws.*

So let's get started!

The equation of state To a very good approximation, for any gas that isn't very dense

$$pV = Nk_BT \tag{3}$$

where N is the number of molecules and k_B is Boltzmann's constant, which showed up in our definition of entropy. You will also see chemists write this as

$$pV = nRT \tag{4}$$

where n is the number of *moles* of molecules. A "mole" is like a dozen. It's a dimensionless unit of number. R is called the ideal gas constant, but it's really just Boltzmann's constant divided by

Avogadro's number, which is itself the value of a mole. Most physicists prefer to just use numbers to represent numbers, instead of numbers of moles.

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Figure 2: The temperature plotted for $N = 0.25 \times 10^{23} = 0.04$ mol. This is one liter of gas at room temperature and pressure.

The internal energy For a monatomic gas (i.e. just one atom per molecule), like argon, to a very good approximation

$$U = \frac{3}{2} N k_B T \tag{5}$$

provided the gas is not very dense, and it is also not too hot. If you had a dense gas, the internal energy would also depend on the volume, and if you had a gas hot enough to excite electronic energy levels, then the temperature dependence would be more complicated.

To summarize, as the temperature of a gas increases or decreases, we know that the internal energy is doing the same.

Detective work Sherlock finds a clue about the financial activity in the suspect's bank account:

Starting balance	$700,000
Ending balance	$600,000
Electronic deposits	+$100,000
Electronic withdrawals	$0
Cash deposits	
Cash withdrawals	

Alas, the receipt had a corner torn, so he cannot read the cash deposits or withdrawals!

What was the sum of the cash deposits and cash withdrawals?

1. -$200,000
2. $0
3. +$200,000

In our case, we will be looking at

ΔU A change in the account balance of internal energy

Q deposit or withdrawal of energy

W deposit or withdrawal of energy

(Note: this is just applying the First Law again.) We can apply these ideas to the rectangle-shaped gas cycle that you have already studied, to find the efficiency of this cycle.

Let's assume a monatomic gas with $N = 0.25 \times 10^{23}$ atoms

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Figure 3: Gas engine path

Time period	W	Q	$\Delta U = Q + W$
0-10s	0		*fill this in first.
10-20s	-100 J		
20-30s	0		
30-40s	50 J		

0-10 s

$$U_i = \frac{3}{2} N k_B T_i \tag{6}$$
$$p_i V_i = N k_B T_i \tag{7}$$
$$U_i = \frac{3}{2} p_i V_i \tag{8}$$
$$= \frac{3}{2}[10^5 \text{ Pa}][0.5 \times 10^{-3} \text{ m}^3] \tag{9}$$
$$= 75 \text{ J} \tag{10}$$
$$U_f = \frac{3}{2}[2 \times 10^5 \text{ Pa}][0.5 \times 10^{-3} \text{ m}^3] \tag{11}$$
$$= 150 \text{ J} \tag{12}$$

From this we can find the Q from the First Law

$$\Delta U = Q + W \tag{13}$$
$$75 \text{ J} = Q + 0 \tag{14}$$
$$Q = 75 \text{ J} \tag{15}$$

10-20 s

$$U_i = \frac{3}{2} N k_B T_i \tag{16}$$
$$p_i V_i = N k_B T_i \tag{17}$$
$$U_i = \frac{3}{2} p_i V_i \tag{18}$$
$$= \frac{3}{2}[2 \times 10^5 \text{ Pa}][0.5 \times 10^{-3} \text{ m}^3] \tag{19}$$
$$= 150 \text{ J} \tag{20}$$
$$U_f = \frac{3}{2}[2 \times 10^5 \text{ Pa}][10^{-3} \text{ m}^3] \tag{21}$$
$$= 300 \text{ J} \tag{22}$$

$$\Delta U = Q + W \tag{23}$$
$$150 \text{ J} = Q - 100 \text{ J} \tag{24}$$
$$Q = 250 \text{ J} \tag{25}$$

20-30 s

$$\begin{align} \Delta U &= \frac{3}{2}(p_f V_f - p_i V_i) \tag{26}\\ &= \frac{3}{2}(10^5 \text{ Pa} - 2\times 10^5 \text{ Pa})[10^{-3} \text{ m}^3] \tag{27}\\ &= -150 \text{ J} \tag{28}\\ Q &= -150 \text{ J} \tag{29} \end{align}$$

30-40 s

$$\begin{align} \Delta U &= \frac{3}{2}(p_f V_f - p_i V_i) \tag{30}\\ &= \frac{3}{2}[10^5 \text{ Pa}](0.5\times 10^{-3} \text{ m}^3 - 10^{-3} \text{ m}^3) \tag{31}\\ &= -75 \text{ J} \tag{32}\\ Q &= -75 \text{ J} - 50 \text{ J} \tag{33}\\ &= -125 \text{ J} \tag{34} \end{align}$$

Efficiency Taken together, this gives us To find the efficiency of this cycle, we just need the net heat and the sum of all the energy *added by heating.*

Time period	W	Q	$\Delta U = Q + W$
0-10s	0	75 J	75 J
10-20s	-100 J	250 J	150 J
20-30s	0	-150 J	-150 J
30-40s	+50 J	-125 J	-75 J

$$\begin{align} W_{\text{net}} &= -100 \text{ J} + 50 \text{ J} \tag{35}\\ &= -50 \text{ J} \tag{36} \end{align}$$

Now the sign of this is negative, meaning the gas *lost* this much energy due to work done. When we discussed heat engines, we defined work as a positive number when the engine did work, so we'll have to flip the sign.

To find the Q_{in} we just have to add up the positive Q values:

$$\begin{align} Q_{\text{in}} &= 75 \text{ J} + 250 \text{ J} \tag{37}\\ &= 325 \text{ J} \tag{38} \end{align}$$

This gives us an efficiency of

$$\begin{align} \text{efficiency} &= \frac{|W_{\text{net}}|}{Q_{\text{in}}} \tag{39}\\ &= \frac{50 \text{ J}}{325 \text{ J}} \tag{40}\\ &= 15\% \tag{41} \end{align}$$

This doesn't sound great. But before we conclude that this is a terrible engine, we should check its Carnot efficiency, to see whether our rectangular cycle was a bad idea, or if we just need a hotter hot reservoir or maybe a colder cold reservoir.

Carnot efficiency

$$\text{Carnot efficiency} = 1 - \frac{T_C}{T_H} \tag{42}$$

To find the cold and hot temperatures, we can consult the temperature plot (or the ideal gas equation of state), and see that the left bottom corner is coldest, and the right top corner is hottest.

$$\begin{align}
T_C &= \frac{[1\text{ atm}][0.5\text{ L}]}{Nk_B} \tag{43}\\
T_H &= \frac{[2\text{ atm}][1\text{ L}]}{Nk_B} \tag{44}\\
\frac{T_C}{T_H} &= \frac{0.5}{2} \tag{45}\\
&= 0.25\% \tag{46}\\
\text{Carnot efficiency} &= 75\% \tag{47}
\end{align}$$

So we totally could have done better. You get to attempt another cycle in `https://paradigms.oregonstate.edu/activity/748`, to see if you can do better that way.