Wind gives very low cost, once you have built a wind turbine.

In `https://paradigms.oregonstate.edu/activity/719` we worked out the kinetic energy of generated wind in order to find the consumptions of a car. In this activity we will look at capturing the kinetic energy of wind.

Your task will be to estimate the power output of the Biglow Canyon Wind Farm in Oregon. This farm consists of 225 wind turbines. There are three arms on each wind turbine, each arm having a length of 46 meters!

You can think of a wind turbine as a magic disk that captures the kinetic energy of the air that passes through the circle formed by the turbine. **Suppose would we extract all the K.E. from the incoming air given the current wind speed at Biglow Canyon. Estimate how much energy would we get per second. How does this compare with the rated capacity for Biglow Canyon Wind Farm of 450 MJ/s?**

Solution Let's call v_i the speed of the wind that the turbine is harvesting, and the cross-sectional area of the turbine A. In time Δt, the volume of air passing through the turbine would be

$$\text{volume of air} = v_i \Delta t A \tag{1}$$

The mass of this air comes from multiplying by the density. Then the kinetic energy is

$$\begin{aligned}
\text{K.E. passing} &= \frac{1}{2}mv^2 && (2)\\
&= \frac{1}{2}(\rho_{\text{air}} v_i \Delta t A) v_i^2 && (3)\\
&= \frac{1}{2}\rho_{\text{air}} A v_i^3 \Delta t && (4)
\end{aligned}$$

From this we can see that the *rate* of energy produced will be

$$\text{rate of energy production} = \frac{1}{2}\rho_{\text{air}} A v_i^3 \tag{5}$$

So now we just need to put in numbers. If the current wind speed is 32 mph (14 m/s) then we get

$$\begin{aligned}
\text{rate of energy production from one turbine} &= \frac{1}{2}[1.2\ \text{kg/m}^3]\pi\,[46\ \text{m}]^2\,[14\ \text{m/s}]^3 && (6)\\
&\approx 10^{-0.3}10^{0.1}10^{0.5}\left[10^{1.7}\right]^2\left[10^{1.2}\right]^3\ \text{J/s} && (7)\\
&= 10^{7.3}\ \text{J/s} && (8)\\
&= 2\times 10^{7}\ \text{J/s} && (9)\\
&= 20\ \text{MJ/s} && (10)
\end{aligned}$$

This is already quite promising, in terms of the rated production value, because this is for a single turbine. To get the production of the entire wind farm, we have to multiply by the 200 turbines.

$$\text{rate of energy production from the farm} = 200 \cdot 2 \times 10^7 \text{ J/s} \quad (11)$$
$$= 4 \times 10^9 \text{ J/s} \quad (12)$$
$$= 4,000 \text{ MJ/s} \quad (13)$$

This about an order of magitude greater than the rated power of the wind farm.

Other wind speeds I used a rather high wind speed for this estimate. If the current wind speed is less, you will get *far* less power, since the power scales as the speed cubed. This makes it very worthwhile to place wind farms in locations with consistent high speed wind.

0.1 Making it more realistic

Our answer above is assuming that we can turn *all* the energy from the air into electrical energy. **Brainstorm reasons why we might get less energy out.**

Solution We can't actually get all the energy out of the air, or the air would accumulate at our wind turbine, since its speed would be zero. In reality, because the mass of air is conserved (as well as its energy) we must allow the air to keep some kinetic energy so it can get out of the way of more air coming in.

Betz's calculation (and that of two other scientists independently) (1919) found the optimal velocities for a turbine (Betz's Law). The theoretical fraction of the energy that you're able to extract from the air turns out to be about 59%. In practice, turbines tend to capture 45%-50% of the incoming energy. If we decrease our estimate by a factor of 2 above, we find a result that is still 5 times greater than the rated capacity. On the day I am writing this (April 5, 2021), the peak wind speed is 7 m/s, and the lowest is 3 m/s. So presumably they are rating it for lower wind speeds.

The optimal scenario turns out to be that the air actually passing through the turbine is at $\frac{2}{3}v_i$, and the air leaving the turbine is at $v_f = \frac{1}{3}v_i$. In this

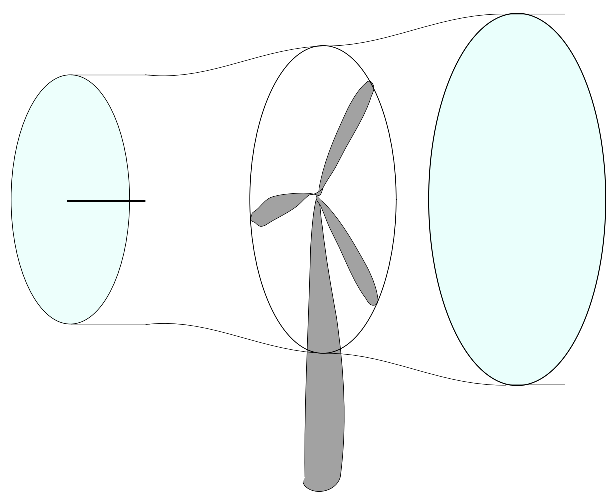

Figure 1: The initial area of the air is $A_i = \frac{2}{3}A$, where A is the area of the turbine, and the final area of the air column is $A_f = 2A$. These areas arise from the volume of the air moving in a given time Δt not changing, while the speed does change.

case, the energy produced is given by

$$\text{energy produced in time } \Delta t = \frac{1}{2}\rho_{\text{air}} A \left(\frac{2}{3} v_i \Delta t\right) \left(v_i^2 - \left(\frac{1}{3} v_i\right)^2\right) \tag{14}$$

$$= \frac{1}{2}\rho_{\text{air}} A \left(1 - \frac{1}{9}\right) \frac{2}{3} v_i^3 \tag{15}$$

$$= \frac{16}{27}\left(\frac{1}{2}\rho_{\text{air}} A v_i^3\right) \tag{16}$$