

## 1 Cyclic Translation of 3 Distinguishable Beads

Consider 3 (distinguishable beads) arranged clockwise on a ring.

- Construct a matrix operator that translates (moves) each bead clockwise one “bead slot” along the ring.
- Write down an eigenvalue equation for this operator.
- Find eigenvalues and eigenvectors of this operator.

## 2 Adding Operators with a Common Set of Eigenvectors

- Consider linear operators  $\hat{A}$  and  $\hat{B}$  share a complete set of eigenvectors but with different eigenvalues.

$$\begin{aligned}\hat{A}|ab\rangle &= a|ab\rangle \\ \hat{B}|ab\rangle &= b|ab\rangle\end{aligned}$$

Now consider an operator  $\hat{C}$  that is a linear combination of  $\hat{A}$  and  $\hat{B}$ . Show that  $|ab\rangle$  is also an eigenvector of  $\hat{C}$  and find its eigenvalues.

## 3 $\hat{S}^\uparrow$ and $\hat{S}^\downarrow$

Consider the operators:

$$\hat{S}^\uparrow \doteq \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\hat{S}^\downarrow = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- Show that  $\hat{S}^\uparrow$  and  $\hat{S}^\downarrow$  are inverses of each other.
- Show that the eigenvectors of  $\hat{S}^\downarrow$  are also eigenvectors of  $\hat{S}^\uparrow$ .
- Find the three eigenvalues of the operator  $(\hat{S}^\uparrow + \hat{S}^\downarrow)$  and show that at least one of them checks out.