

## 1 Sphere Table

Attached, you will find a table showing different representations of physical quantities associated with a quantum particle confined to a sphere. Fill in all of the missing entries. Hint: You may look ahead. We filled out a number of the entries throughout the table to give you hints about what the forms of the other entries might be. pdf link for the Table or doc link for the Table

## 2 QM Sphere with Time Dependence

Consider a quantum particle on a sphere. At  $t = 0$ , the particle is in state:

$$|\psi(t = 0)\rangle = \frac{1}{\sqrt{2}} (|2, 0\rangle + |1, 0\rangle)$$

Calculate the following quantities for some later time,  $t > 0$ , and identify whether each quantity is time-dependent.

- $|\psi(t)\rangle$ .
- $\langle L_z \rangle$
- $\mathcal{P}(L^2 = 6\hbar^2)$
- The probability that the particle can be found in the “southern” hemisphere.
- The probability that the particle can still (at the time  $t$ ) be found in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|2, 0\rangle + |1, 0\rangle)$$

## 3 Hydrogen, Version 1

A hydrogen atom is initially in the superposition state

$$|\psi(t = 0)\rangle = \frac{1}{\sqrt{14}}|2, 1, 1\rangle - \frac{2}{\sqrt{14}}|3, 2, -1\rangle + \frac{3}{\sqrt{14}}|4, 2, 2\rangle. \quad (1)$$

- What are the possible results of a measurement of the energy and with what probabilities would they occur? Plot a histogram of the measurement results. Calculate the expectation value of the energy.
- What are the possible results of a measurement of the angular momentum operator  $L^2$  and with what probabilities would they occur? Plot a histogram of the measurement results. Calculate the expectation value of  $L^2$ .
- What are the possible results of a measurement of the angular momentum component operator  $L_z$  and with what probabilities would they occur? Plot a histogram of the measurement results. Calculate the expectation value of  $L_z$ .
- How do the answers to (a), (b), and (c) depend upon time?