1 Properties of Legendre Polynomials

(a) Use technology such as *Mathematica* or *Maple* or *Python* to find the first 5 Legendre polynomials. This question is simply asking you to find the command in your preferred computer algebra system and learn the syntax to call the polynomials.

(b) Use Rodrigues' formula to calculate the first 3 Legendre polynomials. (You may use computer technology like *Mathematica* or *Maple* or *Python* to help with the derivatives. This question is asking you to find Rodrigues' formula (Googling it is fine) and learn how to use it to generate the Legendre polynomials.)

2 Legendre Polynomial Series for the Sine Function

Use your favorite technology tool (*e.g.* Maple, Mathematica, Matlab, Python, pencil) to generate the Legendre polynomial expansion to the function $f(z) = \sin(\pi z)$. How many terms do you need to include in a partial sum to get a "good" approximation to $f(z)$ for $-1 < z < 1$? What do you mean by a "good" approximation? How about the interval $-2 < z < 2$? How good is your approximation? Discuss your answers. Answer the same set of questions for the function $g(z) = \sin(3\pi z)$

3 Laplace's Equation in Polar Coordinates

(a) Write down Laplace's equation in two dimensions in polar coordinates.

(b) Use the separation of variables procedure to separate this partial differential equation into two ordinary differential equations.

(c) Write down a complete set of eigenstates of the ϕ equation. Justify your answer. You do not NEED to calculate anything here, but if you quote some answer that you already know, say how/where you know the answer. DO NOT TRY TO SOLVE THE r EQUATION!

4 Chain rule for changing 1 independent variable

(a) Use the chain rule to show that $\frac{d}{dt} = v\frac{d}{dx}$.

(b) A point particle moving along the x-axis with an initial speed $v_0 \neq 0$ is subject to a linear drag force as described by the equation: $\frac{dv}{dt} = -\frac{b}{m}v$, where b and m are constants. Find $v(x)$.

(c) Use the chain rule to represent the operator $\frac{d}{d\theta}$ in terms of the cartesian coordinate z on the unit sphere.

- Hint 1: Draw a picture and think triangles.
- Hint 2: Be sure to substitute all the θ's for z's.