

1 Reflection and Transmission at a Sharp Boundary

A traveling wave in a string is incident from the left (L) and propagates at speed v_1 , encountering a second string at $x = 0$. It partially reflects, returning in the same string with speed v_1 . It partially continues into a second string, traveling with speed v_2 .

$$\psi(x, t) = \begin{cases} \psi_L(x, t) = \operatorname{Re}[Ae^{i(-\omega_1 t + k_1 x)}] + \operatorname{Re}[Be^{i(-\omega_1 t - k_1 x)}] & \text{for } x \leq 0 \\ \psi_R(x, t) = \operatorname{Re}[Ce^{i(-\omega_2 t + k_2 x)}] & \text{for } x \geq 0 \end{cases}$$

- (a) Explain the meanings of the terms in the wavefunctions and say which directions the waves propagate.
- (b) Explain why $\omega_1 = \omega_2 = \omega$
- (c) What equation represents the statement, “The string must be continuous at the boundary”? Show that it leads to $A + B = C$.
- (d) Write the piecewise function for $\frac{\partial \psi(x, t)}{\partial x}$.
- (e) What equation represents the statement, “The transverse component of the force at the boundary must sum to zero”? Show that it leads to $k_1 A - k_1 B = k_2 C$
- (f) Solve the equations in (c) and (e) and find the displacement reflection and transmission coefficients $R_\psi \equiv \frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}$ and $T_\psi \equiv \frac{C}{A} = \frac{2k_1}{k_1 + k_2}$.
- (g) Look carefully at the expression in (d) and show that you can define a reflection coefficient for $\frac{\partial \psi}{\partial x}$ and that it is $R_{\frac{\partial \psi}{\partial x}} = \frac{k_2 - k_1}{k_1 + k_2}$
- (h) Look carefully at the expression in (d) and show that you can define a transmission coefficient for $\frac{\partial \psi}{\partial x}$ and that it is $T_{\frac{\partial \psi}{\partial x}} = \frac{2k_2}{k_1 + k_2}$

2 Light Propagation in a Vacuum

None Light propagates in vacuum with speed c and in a medium with speed $v = \frac{c}{n}$ where n is the refractive index ($n > 1$). (The refractive index is not an integer!) Show that when light is incident from vacuum ($n = 1$) onto glass ($n = 1.5$), about 4% of the energy (which is proportional to $|\psi|^2$) is reflected. Also show that the light changes its phase angle by π when it is reflected.

3 Square Wave in a Rope

None A rectangular traveling pulse is launched with speed v_1 from the left into a very long rope. At the boundary at $x = 0$, the pulse is partially reflected and partially transmitted into a second rope where the transmitted pulse moves with velocity v_2 . The reflected (black) and transmitted (red) pulses are depicted some time after the original pulse encounters the boundary. The system obeys the NDWE.

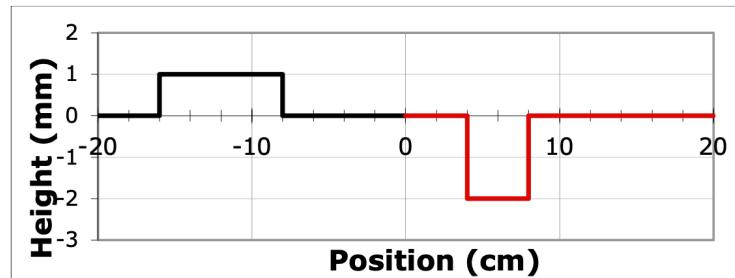


Figure: A point in time of the graph of a square wave after reflecting and transmitting at $x = 0$.

- (a) Use the widths of the reflected and transmitted pulses to find the ratio of v_1 to v_2 . Explain.
- (b) Calculate the relative velocities using the pulse locations and show that this is consistent with (a).
- (c) Is the mass density of the red rope smaller or larger than the black rope? Why?
- (d) Describe the original pulse (height, polarity, length), showing qualitative and quantitative reasoning.
- (e) Can you determine the velocities v_1 and v_2 ? If so, what are they, and if not, what information would you need?