Due 8pm Tuesday evening (contact Ethan if you need an extension).

1 Using Gibbs Free Energy

You are given the following Gibbs free energy:

$$G = -kTN \ln\left(\frac{aT^{5/2}}{p}\right),$$

where a is a constant (whose dimensions make the argument of the logarithm dimensionless).

(a) Compute the entropy.

(b) Work out the heat capacity at constant pressure C_p.

(c) Find the connection among V, p, N, and T, which is called the equation of state (Hint: find the volume as a partial derivative of the Gibbs free energy). Simplify the final expression as much as possible.

(d) Find the internal energy U from the expression for G that you were given in the main prompt. Simplify the final expression as much as possible.

2 Ideal gas internal energy

In this problem, you will prove that the internal energy of an ideal gas depends on temperature, but not on volume, based soley on the ideal gas equation:

$$pV = Nk_BT \tag{1}$$

and of course your knowledge of thermodynamics. It's a pretty tricky proof, so I'll step you through it.

(a) To begin with, use the Helmholtz free energy $F = U - TS$ to show that

$$\left(\frac{\partial U}{\partial V}\right)_T = -p + T\left(\frac{\partial S}{\partial V}\right)_T \tag{2}$$

for *any* material.

(b) Now show that for any material

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V. \tag{3}$$

(c) Finally, show that for an ideal gas

$$\left(\frac{\partial U}{\partial V}\right)_T = 0. \tag{4}$$

Remember that the only statement we can assume about the ideal gas is $pV = Nk_BT$. We have not been given an expression for U.

3 Non-Ideal Gas

The equation of state of a gas that departs from ideality can be approximated by

$$p = \frac{NkT}{V}\left(1 + \frac{NB_2(T)}{V}\right),$$

where B_2 is called the second virial coefficient. B_2 is a function of T, so it is usually written as $B_2(T)$. The function $B_2(T)$ increases monotonically with temperature. Find $\left(\frac{\partial U}{\partial V}\right)_T$ and determine its sign.

4 Plastic Rod

When stretched to a length L the tension force τ in a plastic rod at temperature T is given by its Equation of State

$$\tau = aT^2(L - L_o)$$

where a is a positive constant and L_o is the rod's unstretched length. For an unstretched rod (i.e. $L = L_o$) the heat capacity at constant length is $C_L = bT$ where b is a constant. Knowing the internal energy at T_o, L_o (i.e. $U(T_o, L_o)$) find the internal energy $U(T_f, L_f)$ at some other temperature T_f and length L_f.

(a) (1 point) Write an expression for the exact differential dU in terms of dT and dL (we've been calling this type of expression an "overlord equation").

(b) Show that the partial derivative $(\partial U/\partial L)_T = -aT^2(L - L_o)$.

(c) Integrate dU *very carefully* in the $T - L$ plane, keeping in mind that $C_L = bT$ holds *only* at $L = L_o$ to find $U(T_f, L_f) - U(T_0, L_0)$.