1 Angles from the Dot Product

Let

$$\vec{\boldsymbol{a}} = \hat{\boldsymbol{x}} - 3\hat{\boldsymbol{y}} - \hat{\boldsymbol{z}}$$
$$\vec{\boldsymbol{b}} = \hat{\boldsymbol{x}} + \hat{\boldsymbol{y}} + 2\hat{\boldsymbol{z}}$$
$$\vec{\boldsymbol{c}} = -2\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}} + \hat{\boldsymbol{z}}$$
$$\vec{\boldsymbol{d}} = -\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}} + \hat{\boldsymbol{z}}$$

Use the dot product to determine: Which pairs of these vectors (if any)

(a) Are perpendicular?

(b) Are parallel?

(c) Have an angle less than $\pi/2$ between them?

(d) Have an angle of more than $\pi/2$ between them?

2 Tetrahedron

Using a dot product, find the angle between any two line segments that join the center of a regular tetrahedron to its vertices. *Hint:* Think of the vertices of the tetrahedron as sitting at the vertices of a cube (at coordinates (0,0,0), (1,1,0), (1,0,1) and (0,1,1)—you may need to build a model and play with it to see how this works!)

3 Graphs Involving the Distance Formula

Learn more about the geometry of $|\vec{r} - \vec{r'}|$ in two dimensions.

(a) Make sketches of the following functions, by hand, on the same axes:

$$y = \sin x$$
$$y = \sin(x + 2)$$

Briefly describe the role that the number 2 plays in the shape of the second graph

(b) Make a sketch of the graph

$$|\vec{r} - \vec{a}| = 2$$

for each of the following values of $\vec{a}$:

$$\vec{a} = \vec{0}$$
$$\vec{a} = 2\hat{x} - 3\hat{y}$$
$$\vec{a} = \text{points due east and is 2 units long}$$

(c) Derive a more familiar equation equivalent to

$$|\vec{r} - \vec{a}| = 2$$

for arbitrary $\vec{a}$, by expanding $\vec{r}$ and $\vec{a}$ in rectangular coordinates. Simplify as much as possible. (Ok, ok, I know this is a terribly worded question. What do I mean by "more familiar"? What do I mean by "simplify as much as possible"? Why am I making you read my mind? Try it anyway. Real life is not full of carefully worded problems. Bonus points to anyone who can figure out a better way of wording the question that doesn't give the point away.)

(d) Write a brief description of the geometric meaning of the equation

$$|\vec{r} - \vec{a}| = 2$$