

1 Constant Acceleration by Separation of Variables

(a) **Calculate:** Treat Newton's 2nd law as a separable differential equation and solve for the velocity and position as a function of time of an object that is all of the following:

- moving in one dimension,
- not initially at the origin of coordinates,
- moving with a non-zero initial speed,
- experiences a constant force.

(b) **Reflect:** Do your answers look familiar? If yes, from where? If not, how would you have to modify these equations to be similar to equations you know?

2 Separable ODE linear + constant

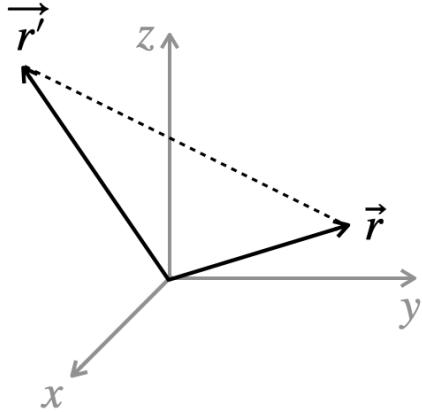
Solve the differential equation:

$$\frac{dv}{dt} = -b - cv \text{ where } v(t = 0) = v_0$$

3 Distance Formula in Curvilinear Coordinates

The distance $|\vec{r} - \vec{r}'|$ between the point \vec{r} and the point \vec{r}' is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r} - \vec{r}'|$ between the point \vec{r} and the point \vec{r}' in rectangular coordinates.



(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}'| = \sqrt{s^2 + s'^2 - 2ss' \cos(\phi - \phi') + (z - z')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}'| = \sqrt{r'^2 + r^2 - 2rr' [\sin \theta \sin \theta' \cos(\phi - \phi') + \cos \theta \cos \theta']}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(d) Now assume that \vec{r}' and \vec{r} are in the x - y plane. Simplify the previous two formulas.