# 1 Bead on a Spinning Wire Hoop

(*modified from Taylor Ex. 7.6*)

A small bead of mass $m$ is threaded on a frictionless circular wire hoop of radius $R$. The hoop lies in a vertical plane, which is forced to rotate about the hoop's vertical diameter with constant angular velocity $\dot{\phi} = \omega$, as shown in Figure 7.9. The bead's position on the hoop is specified by the angle $\theta$ measured up from vertical.

(a) Write down the Lagrangian for the system in terms of the generalized coordinate $\theta$ and find $\ddot{\theta}$. Discuss at least three strategies for making sense of your answer.

(b) Find the angles for which $\ddot{\theta} = 0$ - these are equilibrium angles. Show these locations on a sketch of the hoop. Use at last three strategies for making sense of your answer. Include and discuss a plot the equilibrium angles vs. $\omega$

# 2 A Ball Confined to the Surface of a Sphere in Near-Earth Gravity

A ball with mass $m$ is confined to move on the surface of a sphere with radius $r = R$. A convenient choice of coordinates is spherical, $r$, $\theta$, $\phi$, with the polar axis pointing straight down. (Remember: in physics, $\phi$ is the azimuthal angle in the $xy$-plane and $\theta$ is the angle with the $z$-axis.)

(a) Find the equations of motion using a Lagrangian approach. Use at least three sense-making strategies to evaluate each equation.

(b) Explain what the $\phi$ equation indicates about the $z$-component of angular momentum.

(c) Discuss the specific special case that $\phi$ = constant. What does the equation of motion for $\theta$ indicate?