1 Speed Squared in Curvilinear Coordinates

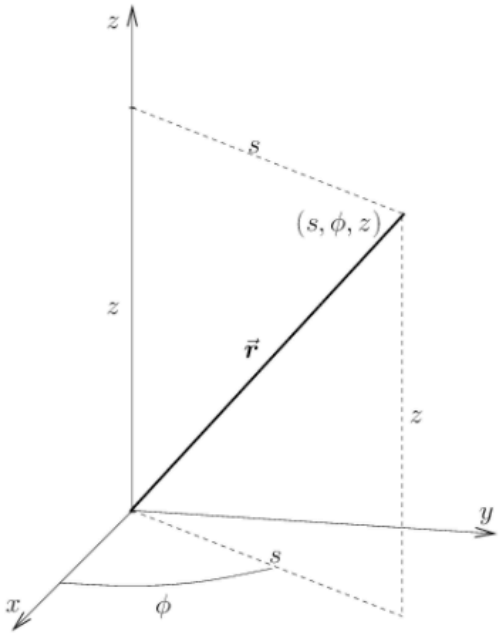
Show that the speed squared written in different standard coordinates systems is:

(a) in rectangular coordinates: $v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$

(b) in cylindrical coordinates: $v^2 = \dot{s}^2 + s^2\dot{\phi}^2 + \dot{z}^2$

(c) in spherical coordinates: $v^2 = \dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2$

where:

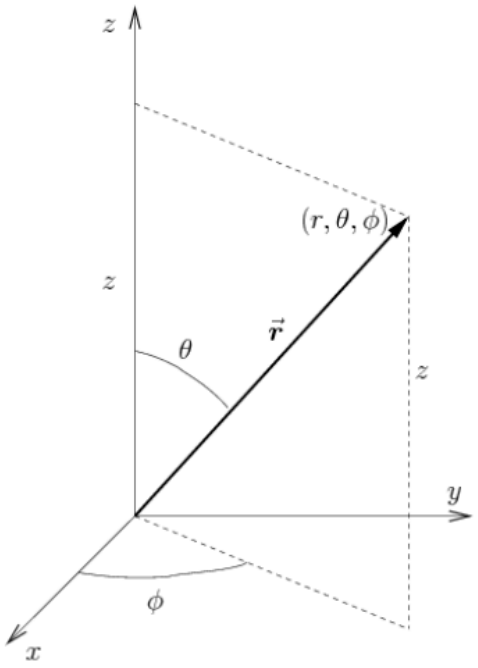


Cylindrical Coordinates

$x = s\cos\phi$

$y = s\sin\phi$

$z = z$



Spherical Coordinates

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

The algebra for these calculations is a little tedius, but it's good to do them once in your career to convince yourself that these expressions are true!

See: GMM: Curvilinear Coordinates for more discussion of curvilinear coordinates. Note that the labels for the aximuthal and polar coordinates in different for math and physics (ϕ and θ are swapped.)