1 Intro to Hyperbolic Trig

(*modified from Taylor 2.33 & 2.34*)

Hyperbolic trigonometry is going to be useful for us both for describing the motion of objects subject to quadratic drag and also for doing special relativity. Familiarity with hyperbolic trig functions will be useful.

The hyperbolic functions $\cosh z$ and $\sinh z$ are defined as follows:

$$\cosh z = \frac{e^z + e^{-z}}{2} \tag{1}$$

$$\sinh z = \frac{e^z - e^{-z}}{2} \tag{2}$$

$$\tag{3}$$

for any z, real or complex.

(a) Plot the behavior of $\cosh z$ and $\sinh z$ over a suitable range of real values z. Describe the behavior of these functions in words and note what the values of the functions are near $z = 0$ and $z = \pm\infty$.

(b) Show that $\cosh^2 z - \sinh^2 z = 1$

(c) Calculate the derivatives of $\cosh z$ and $\sinh z$. Do the derivatives make sense when looking at the plots of the originals functions? Explain

(d) Show that $\int \left(1/\sqrt{1+x^2}\right) dx = \operatorname{arcsinh} x$
(Hint: One way to do this is to make the substitution $x = \sinh z$)

2 More Hyperbolic Trig

The hyperbolic function $\tanh z$ is defined as $\tanh z = \sinh z / \cosh z$.

(a) Plot the behavior of this function over a suitable range of real values z. Describe the behavior of the function in words and note what the values of the functions are a $z = 0$ and $z = \pm\infty$.

(b) Show that the derivative of $\tanh z = \operatorname{sech}^2 z$, where $\operatorname{sech} z$ is the hyperbolic secant.

(c) Show that $\int \tanh z \, dz = \ln \cosh z$.

(d) Show that:

$$\int \frac{dx}{1 - x^2} = \operatorname{arctanh} x$$

(Hint: One way to do this is to make the substitution $x = \tanh z$).

3 Velocity with Linear Air Drag

You are at a park tossing a small ball to a friend. The ball has a mass m and you toss it with an initial $\vec{v}_0 = v_{0,x}\hat{x} + v_{0,y}\hat{y}$ into the air so that your friend standing some distance away will catch it. The mass experiences a drag force from air that is linear with its velocity.

(a) Find the velocity of the ball as a function of time.

(b) *Sensemaking: Check the Dimensions* Do the dimensions of your equation balance and make sense? Explain.

(c) *Sensemaking: Examine the Behavior of Functions* Plot the components of the ball's velocity a function of time. Do your plots make conceptual sense given the physical situation? Explain.

(d) *Sensemking: Evaluate Special Cases* Does your velocity equation make sense at $t = 0$ and at large t. Explain. (Try to identify what you expect the velocity to be in these cases before you plug into your answer. Be sure to state whether your answer agrees with your expectation.)

4 Position with Linear Air Drag

Consider the same situation as in "Velocity with Linear Air Drag".

(a) Find the position of the ball as a function of time using your answer to the previous problem.

(b) *Sensemaking: Check the Dimensions:* Do the dimensions of your equation balance and make sense? Explain.

(c) *Sensemaking: Examine the Behavior of Functions* Plot the components of the ball's position. Does your plot make conceptual sense given the physical situation? Explain.

(d) *Sensemaking: Evaluate Special Cases* Does your answer make sense at $t = 0$ and at large t. Explain. (Try to identify what you expect the velocity to be in these cases before you plug into your answer. Be sure to state whether your answer agrees with your expectation.)