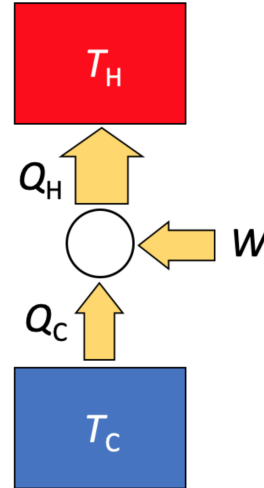


1 Heat Pump

The diagram shows a machine (the white circle) that moves energy from a cold reservoir to a hot reservoir. We will consider whether a machine like this is useful for heating a family home in the winter when the temperature inside the family home is T_H , and the temperature outside the house is T_C . To quantify the performance of this machine, I'm interested in the ratio Q_H/W , where Q_H is the heat energy entering the house, and W is the net energy input in the form of work. (W is the energy I need to buy from the electricity company to run an electric motor). Starting from the 1st and 2nd laws of thermodynamics, find the maximum possible value of Q_H/W . This maximum value of Q_H/W will depend solely on the ratio of temperatures T_H and T_C .

Sensemaking: Choose realistic values of T_H and T_C to describe a family home on a snowy day. Based on your temperature estimates, what is the maximum possible value of Q_H/W ?



2 Water and air heat capacity

In the last homework, you looked up the specific heat capacity of water and air ($c_{p,\text{water}} = 4.2 \text{ J/(g.K)}$ and $c_{p,\text{air}} = 1.0 \text{ J/(g.K)}$) to analyze changes in the earth's climate. In this problem, you'll estimate $c_{p,\text{water}}$ and $c_{p,\text{air}}$ from first principles. Note: You'll use the equipartition theorem which is a coarse-grained alternative to using the full machinery of statistical mechanics, so, the answers might be off by a few %.

- (a) For liquid water at room temperature, treat every oxygen atom and hydrogen atom as a point mass held in place by a 3-dimensional network of springs. These "springs" arise from intra-molecular bonds (bonds within an H_2O molecule) and inter-molecular forces (the forces between neighboring H_2O molecules). Assume 1 gram of water and calculate the total number of degrees of freedom. Then find the total internal energy as a function of temperature, and the specific heat capacity at constant volume. Compare with measured value of $c_{p,\text{water}}$. Note: Liquid water doesn't expand/contract very much when heated, so $c_{p,\text{water}} \approx c_{v,\text{water}}$.
- (b) The main components of air are nitrogen and oxygen. They are both diatomic gas molecules. You can model an O_2 gas molecule, or N_2 gas molecule, as two point masses connected by a stiff spring. The spring is so stiff that the energy quanta needed to excite the spring is bigger than $k_B T/2$ when $T = 300 \text{ K}$. Therefore, you can treat the spring like a rigid rod. Calculate the total number of degrees of freedom in 1 gram of air. Then find the total internal energy as a function of temperature, and the specific heat capacity at constant volume. Compare with measured value of $c_{v,\text{air}} = 0.717 \text{ J/(g.K)}$. Note, $c_{p,\text{air}}$ is about 40% higher than $c_{v,\text{air}}$ because air held at constant pressure converts a sizable fraction of the heat into work when the gas expands.