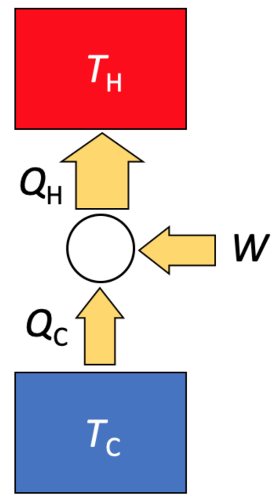
1 Heat Pump

The diagram shows a machine (the white circle) that moves energy from a cold reservoir to a hot reservoir. We will consider whether a machine like this is useful for heating a family home in the winter when the temperature inside the family home is T_{H}, and the temperature outside the house is T_{C}. To quantify the performance of this machine, I'm interested inthe ratio Q_{H}/W, where Q_{H} is the heat energy entering the house, and W is the net energy input in the form of work. (W is the energy I need to buy from the electricity company to run an electric motor). Starting from the 1^{st} and 2^{nd} laws of thermodynamics, find the maximum possible value of Q_{H}/W. This maximum value of Q_{H}/W will depend solely on the ratio of temperatures T_{H} and T_{C}.



Sensemaking: Choose realistic values of T_{H} and T_{C} to describe a family home on a snowy day. Based on your temperature estimates, what is the maximum possible value of Q_{H}/W?

2 Water and air heat capacity

In the last homework, you looked up the specific heat capacity of water and air ($c_{\mathrm{p,water}} = 4.2$ J/(g.K) and $c_{\mathrm{p,air}} = 1.0$ J/(g.K)) to analyze changes in the earth's climate. In this problem, you'll estimate $c_{p,\mathrm{water}}$ and $c_{p,\mathrm{air}}$ from first principles. Note: You'll use the equipartition theorem which is a coarse-grained alternative to using the full machinary of statistical mechanics, so, the answers might be off by a few %.

(a) For liquid water at room temperature, treat every oxygen atom and hydrogen atom as a point mass held in place by a 3-dimensional network of springs. These "springs" arise from intra-molecular bonds (bonds within an H_2O molecule) and inter-molecular forces (the forces between neighboring H_2O molecules). Assume 1 gram of water and calculate the total number of degrees of freedom. Then find the total internal energy as a function of temperature, and the specific heat capacity at constant volume. Compare with measured value of $c_{\mathrm{p,water}}$. Note: Liquid water doesn't expand/contract very much when heated, so $c_{\mathrm{p,water}} \approx c_{\mathrm{v,water}}$.

(b) The main components of air are nitrogen and oxygen. They are both diatomic gas molecules. You can model an O_2 gas molecule, or N_2 gas molecule, as two point masses connected by a stiff spring. The spring is so stiff that the energy quanta needed to excite the spring is bigger than $k_{\mathrm{B}}T/2$ when $T = 300$ K. Therefore, you can treat the spring like a rigid rod. Calculate the total number of degrees of freedom in 1 gram of air. Then find the total internal energy as a function of temperature, and the specific heat capacity at constant volume. Compare with measured value of $c_{\mathrm{v,air}} = 0.717$ J/(g.K). Note, $c_{\mathrm{p,air}}$ is about 40% higher than $c_{\mathrm{v,air}}$ because air held at constant pressure converts a sizable fraction of the heat into work when the gas expands.