

1 Pipe flow line integral

For $-3 \leq x \leq 3$ feet, the two-dimensional flow rate in feet per second of storm water in a pipe can be modeled by $6e^{-x^2}\hat{\mathbf{y}}$. For each of the following curves, choose a parametrization, construct a line integral of the given vector field over the curve and determine its (approximate) value.

- $x^2 + y^2 = 4$.
- $y = 2 - x^2$.
- $y = 2x$.

2 Saturn and Titan

Titan is a moon of Saturn orbiting a distance 1221.86×10^3 kilometers away and whose mass is 2.36×10^{-4} of Saturn's mass which is 569×10^{24} kilograms. The potential energy due to gravity is modeled by $\Phi = \frac{-GMm}{r}$ where $G = 6.67 \times 10^{-11} \frac{m^3 s^2}{kg}$, M is the mass of Saturn, m is the mass of an object (such as Titan) and r is the distance between Saturn and the object in orbit.

- Describe the level sets of Φ where m and r are variables. On which level set is Titan?
- Which coordinate system would you choose for calculations in this situation?
- Determine the gradient of Φ . In which direction does it point?