1 Lagrange multipliers

Solve *one* of the following problems using the method of Lagrange multipliers.

(a) Find the maximum and minimum values of $x^2 + y^2 - 2x - 2y$ on the circle of radius $\sqrt{8}$ centered at the origin, that is, on the circle $\{x^2 + y^2 = 8\}$

(b) Find the points on the curve $x^2 + xy + y^2 = 3$ which are closest to and furthest from the origin.

2 Velocity I

An object moving with constant velocity passes through the point $(1, 1, 1)$, then through the point $(2, -1, 3)$ five seconds later. What is its velocity vector? What is its speed? What is its acceleration vector? (Assume the coordinates are given in meters.)

3 Velocity II

Suppose $\vec{\boldsymbol{r}}(t) = 3\cos(\omega t)\,\hat{\boldsymbol{x}} + 3\sin(\omega t)\,\hat{\boldsymbol{y}} + 4\omega t\,\hat{\boldsymbol{z}}$ represents the position of a particle on a curve after t seconds (with distance measured in meters and $\omega > 0$).

(a) Is the particle ever moving downward? If so, when?

(b) When does the particle reach a point 12 meters above the ground?

(c) What is the velocity of the particle when it is 12 meters above the ground? What is its speed?

(d) When it is 12 meters above the ground, the particle leaves the curve and moves along the tangent line to the curve. Find an equation for this tangent line.