

1 The metal bar

A one-meter long metal bar is heated unevenly, with temperature in $^{\circ}\text{C}$ at a distance x meters from one end at time t given by

$$T(x, t) = 100e^{-0.1t} \sin(\pi x)$$

- (a) Calculate $\left. \frac{\partial T}{\partial x} \right|_{x=0.2}$ and $\left. \frac{\partial T}{\partial x} \right|_{x=0.8}$. What are these expressions a function of? What is the practical interpretation (in terms of temperature) of these two partial derivatives? What sign does each derivative have, and what do the signs mean in terms of temperature?
- (b) Calculate $\frac{\partial T}{\partial t}$. What is its sign? What is its interpretation in terms of temperature?

2 Partial derivatives

TRUE or FALSE: *Briefly justify your answer!*

- (a) There is a function $P(x, y)$ such that $\frac{\partial P}{\partial x} = y$ and $\frac{\partial P}{\partial y} = x$.
- (b) There is a function $Q(x, y)$ such that $\frac{\partial Q}{\partial x} = y^2$ and $\frac{\partial Q}{\partial y} = x^2$.
- (c) If $L(x, y)$ satisfies $\frac{\partial L}{\partial x} = a$ and $\frac{\partial L}{\partial y} = b$, with both a and b constant, then $z = L(x, y)$ is the graph of a plane.

3 The heater II (wrapup)

After completing the heater II activity, estimate $\frac{\partial T}{\partial x}$ and $\frac{\partial T}{\partial t}$ 15 hours after the heater is turned on, at a point 11 feet from the corner of the room.