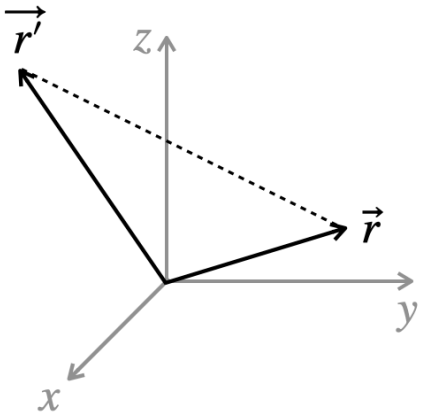
1 Distance Formula in Curvilinear Coordinates

The distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ in rectangular coordinates.



(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{s^2 + s'^2 - 2ss'\cos(\phi - \phi') + (z - z')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{r'^2 + r^2 - 2rr'\left[\sin\theta\sin\theta'\cos(\phi - \phi') + \cos\theta\cos\theta'\right]}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(d) Now assume that $\vec{r}\,'$ and $\vec{r}$ are in the x-y plane. Simplify the previous two formulas.