Submit these problems on Gradescope by 3 pm on Monday 14 October.

1 Tetrahedron

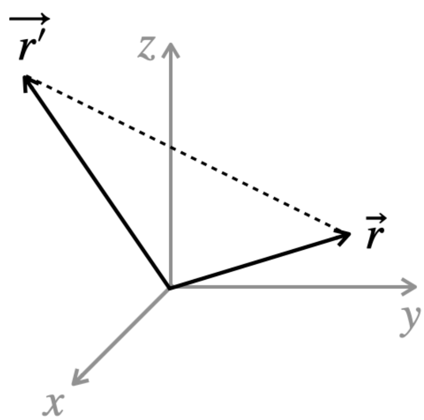
Using a dot product, find the angle between any two line segments that join the center of a regular tetrahedron to its vertices. *Hint:* Think of the vertices of the tetrahedron as sitting at the vertices of a cube (at coordinates (0,0,0), (1,1,0), (1,0,1) and (0,1,1)—you may need to build a model and play with it to see how this works!)

2 Distance Formula in Curvilinear Coordinates

You might want to wait until after Friday's class to do this problem.

The distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ in rectangular coordinates.



(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{s^2 + s'^2 - 2ss'\cos(\phi - \phi') + (z - z')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{r'^2 + r^2 - 2rr'\left[\sin\theta\sin\theta'\cos(\phi - \phi') + \cos\theta\cos\theta'\right]}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(d) Now assume that $\vec{r}\,'$ and $\vec{r}$ are in the x-y plane. Simplify the previous two formulas.

3 Linear Quadrupole (w/ series)

You might want to wait until after Friday's class to do this problem.

Consider a collection of three charges arranged in a line along the z-axis: charges $+Q$ at $z = \pm D$ and charge $-2Q$ at $z = 0$.

(a) Find the electrostatic potential at a point $\vec{r}$ in the xy-plane at a distance s from the center of the quadrupole. The formula for the electrostatic potential V at a point $\vec{r}$ due to a charge Q at the point $\vec{r}'$ is given by:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{|\vec{r} - \vec{r}'|}$$

(b) Assume $s \gg D$. Find the first two non-zero terms of a power series expansion to the electrostatic potential you found in the first part of this problem.

(c) A series of charges arranged in this way is called a linear quadrupole. Why?