

## 1 Matrix Differential Equation I

Let  $P = \begin{pmatrix} i\alpha & 0 \\ 0 & 0 \end{pmatrix}$ ,  $Q = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix}$ ,  $v(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ , and  $c = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ .

(a) Solve the differential equation  $\frac{dv}{dt} = Pv$  with initial conditions  $v(0) = c$ .

(b) Solve the differential equation  $\frac{dv}{dt} = Qv$  with initial conditions  $w(0) = c$ .

*For full credit, your solutions should involve matrix exponentiation.*

## 2 Matrix Differential Equation II

Let  $A = \begin{pmatrix} 0 & \alpha \\ \alpha & 0 \end{pmatrix}$ ,  $v(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ , and  $v_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$ . Solve the differential equation  $\frac{dv}{dt} = Av$  with initial conditions  $v(0) = v_0$ .

*HINT:*  $e^{At} = \begin{pmatrix} \cosh(\alpha t) & \sinh(\alpha t) \\ \sinh(\alpha t) & \cosh(\alpha t) \end{pmatrix}$ .