1 Euler's Formula II

(a) Look up or derive the Taylor series expansions for $\cos\theta$, $\sin\theta$, and e^x.

(b) Show that $e^{i\theta} = \cos\theta + i\sin\theta$.

(c) Verify that $e^{2i\theta} = (e^{i\theta})^2$ using trigonometric identities, that is, show that

$$\cos 2\theta + i\sin 2\theta = (\cos\theta + i\sin\theta)^2$$

2 The Rotation Group $SO(2)$

Consider rotation matrices of the form

$$M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

(a) Show that

  (a) $M(0) = 1$
  (b) $M(-\theta) = M(\theta)^{-1}$
  (c) $M(\alpha+\beta) = M(\alpha)M(\beta)$

(b) Compute $A = \dot{M} = \frac{dM}{d\theta}\Big|_{\theta=0}$.

(c) Show that $M(\theta) = \exp(A\theta)$.